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Improving OFDMA Performance Using Code-Division Multiplexing

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Abstract — This paper analyzes the performance of an orthogonal frequency-division multiple access (OFDMA) scheme in which the carriers assigned to a user are uniformly spaced across the channel bandwidth and code-division multiplexing is used between them. Zero forcing and minimum mean-square error equalization and maximum-likelihood detection are used to compensate for multipath fading. It is found that code-division multiplexing recovers most of the frequency diversity loss inherent to OFDMA. The analyzed scheme makes it possible to benefit from the advantages of OFDMA and approach the performance of MC-CDMA.

Index Terms — Multiple access, OFDMA, MC-CDMA, code-division multiplexing.

INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) has become very popular in wireless communications. After its adoption in the recently developed wireless local area network (LAN) and broadband wireless access (BWA) standards ([1], [2]), this technique is seen today as a strong candidate for future generations of cellular mobile networks. In current systems, OFDM is used with time-division multiple access (TDMA), i.e., users sequentially share the available radio resources and all carriers are assigned to the same user during a given OFDM symbol. A second possibility for the access scheme is to combine OFDM with code-division multiple access (CDMA), a technique that is known as multicarrier CDMA (MC-CDMA) ([3], [4]). A third possibility is to use orthogonal frequency-division multiple access (OFDMA), in which different groups of carriers are assigned to different users.

The basis of OFDMA was described in [5]

and [6]. This technique has a number of interesting features, the basic one for wireless communications systems being that it increases the achievable cell range for a given transmit power. Indeed, the transmit power available at the user terminal in TDMA and CDMA systems is transmitted over the entire channel bandwidth. In contrast, the same power only occupies $(1/M)$ th of this bandwidth in an OFDMA system, which splits the total number of carriers N into M groups of N/M carriers each and assigns these groups to different users. For a given transmit power, such an OFDMA system gains $10\log(M)$ dB in terms of signal-to-noise ratio (SNR) on the uplink, and this translates into a substantial range extension. OFDMA also leads to range extension on the downlink by allocating more power to the group of carriers assigned to distant users and less power to the group of carriers assigned to users that are close to the base station.

While the reduced user signal bandwidth is an advantage for OFDMA in terms of cell range, it increases sensitivity to multipath fading. This is particularly true in OFDMA with clustered carriers, where all carriers assigned to a user may fade simultaneously. In terms of robustness to frequency-selective fading, the second variant of OFDMA, which uses groups of carriers that are regularly spaced across the channel bandwidth, is more appealing. But if uncoded, this variant too is sensitive to multipath fading and requires channel coding to reduce its sensitivity. Therefore, exactly as in OFDM, the conventional approach to combat multipath fading in OFDMA is to use error-correction coding.

In this paper, we investigate another technique to cope with multipath fading in



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spaced-carrier OFDMA. It consists of combining OFDMA with code division-multiplexing (CDM) such that the energy of data symbols is spread across the carriers assigned to the same user. This spreading recovers a great deal of the frequency diversity loss that is inherent to both OFDM transmission and OFDMA-based networks.

The paper is organized as follows: In Section 2, we give a brief review of OFDM. In Section 3 we describe OFDMA and its basic variants. Section 4 describes spaced-carrier OFDMA combined with CDM between the carriers of the same group. In Section 5, we study the performance of this multiple access technique with different types of receivers. Finally, we give our conclusions in Section 6.

A Brief Review of OFDM

In an OFDM system with N carriers, the input data stream is partitioned into N -symbol blocks and the resulting blocks are passed to an N -point inverse DFT operator. A cyclic prefix is inserted between consecutive blocks before subsequent filtering and modulation operations. At the receiver side, the cyclic prefix is dropped and the resulting signal is passed to an N -point forward DFT operator, which converts the signal back to the frequency domain.

The received discrete-time OFDM signal sequence during one symbol period T is

$$\mathbf{r}(i) = \mathbf{H}\mathbf{a}(i) + \boldsymbol{\eta}(i), \quad (1)$$

where i is the index of the transmitted OFDM symbol block and $\boldsymbol{\eta}(i)$ is an additive noise vector. In the sequel, the index i will be dropped for convenience. In equation (1), \mathbf{H} is a diagonal matrix whose elements are given by $[H(e^{j2\pi f_0}), H(e^{j2\pi f_1}), \dots, H(e^{j2\pi(N-1)/N})]$, where

$$H(e^{j2\pi f}) = \sum_{l=0}^L h_l \exp(-j2\pi fl). \quad (2)$$

In (2), the h_l 's are the discrete channel taps of the overall channel impulse response $h(t)$ sampled at the chip rate.

From (1), OFDM needs only a simple multiplication to eliminate channel distortion. Indeed, the optimum solution in the absence

of noise is zero-forcing (ZF) equalization, where the equalization matrix is given by

$$\mathbf{G}_{zf} = \mathbf{H}^\dagger. \quad (3)$$

with $(\cdot)^\dagger$ denoting pseudoinverse.

Since this solution perfectly equalizes the channel frequency response, it is equivalent to forcing to zero the intersymbol interference (ISI) in the time domain. It is known ([7]) that the ML detector for pure OFDM systems is just the ZF equalizer followed by a threshold detector. But ZF equalization enhances noise when the channel has deep notches. To limit noise enhancement and make a trade off between ISI and noise power, minimum mean square error (MMSE) equalization can be used. This criterion minimizes the quantity $E\|\mathbf{G}\mathbf{r} - \mathbf{a}\|^2$ and results in

$$\mathbf{G}_{mmse} = \mathbf{R}_a \mathbf{H}^H (\mathbf{H} \mathbf{R}_a \mathbf{H}^H + \mathbf{R}_\eta)^{-1}, \quad (4)$$

with $\mathbf{R}_a = E[\mathbf{a}\mathbf{a}^H]$ and $\mathbf{R}_\eta = E[\boldsymbol{\eta}\boldsymbol{\eta}^H]$.

When the channel has a deep notch in its frequency spectrum, the overall performance of OFDM will be dictated by that of the symbols transmitted in that region. Suppose that a deep fade occurs on carrier k , in other words, $|H_k| = \min\{|H_k|, i = 1, 2, \dots, N\}$. The equalized k th carrier signal is

$$y_k = r_k / H_k = \mathbf{a}_k + \mathbf{n}_k / H_k. \quad (5)$$

The SNR at the detector input is given by

$$SNR_k = \sigma_a^2 / (\sigma_\eta^2 / |H_k|^2) = |H_k|^2 \sigma_a^2 / \sigma_\eta^2. \quad (6)$$

Averaging the bit error probability over all of the N carriers for the uncoded OFDM system with QPSK signaling gives

$$P_b = \frac{1}{N} \sum_{k=1}^N Q\left(\sqrt{\frac{\sigma_a^2 |H_k|^2}{\sigma_\eta^2}}\right). \quad (7)$$

This expression clearly shows that highly attenuated carriers dictate overall BER performance in uncoded OFDM systems. The classical approach to overcome this problem is to introduce channel coding.



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OFDMA and its Basic Variants

Conventional cellular systems based on OFDM transmission employ time-division multiple access (TDMA). In this scheme, all carriers are assigned to the same user during any OFDM symbol period. This is the case in the IEEE 802.11a wireless LAN standard as well as in the IEEE 802.16a BWA standard. Another possibility is to use MC-CDMA, which spreads data symbols in the time domain, frequency domain or in both. Like OFDM/TDMA, MC-CDMA assigns the entire channel bandwidth to each user.

Another candidate technique is OFDMA, which turns out to be very appealing due to the increased cell coverage that it offers. OFDMA was proposed back in 1996 for interactive services on CATV networks [5] and appears today as one of the potential candidates for future wireless systems. The basis of OFDMA is to assign different carriers to different users. A simple OFDMA scheme consists of partitioning the set of N carriers into M subsets of N/M carriers each and to assign a group of carriers to different users. In this way, resources can be allocated to N/M different users at the same time.

One of the major limitations on the uplink channel in wireless systems is the available transmit power. Compared to TDMA and MC-CDMA, OFDMA increases the cell range on the upstream channel, because it transmits the available power in a small fraction of the channel bandwidth. Assuming that signal attenuation is proportional to the squared distance, an OFDMA system with $M = 8$ will increase the cell coverage on the uplink by 18 dB. Cell range extension can also be achieved on the downlink by allocating more power to the carriers assigned to distant users than to carriers assigned to users that are close to the base station.

The two basic forms of OFDMA are clustered-carrier OFDMA (CC-OFDMA) and spaced-carrier OFDMA (SC-OFDMA). The price paid for increased cell extension of OFDMA is the significant loss of frequency diversity. This is particularly true for CC-OFDMA, because the fade events have a strong correlation between adjacent carriers,

and therefore all carriers of the same group can be faded simultaneously. SC-OFDMA alleviates this problem, because the carriers of the same group span the entire channel bandwidth and fade events are much less correlated than in CC-OFDMA. Still, SC-OFDMA does not completely solve the problem, because the symbols transmitted at faded carriers need to be protected using coding or some other techniques.

Combining SC-OFDMA with Frequency-Domain CDM

An efficient way to cope with multipath fading and recover frequency-diversity in SC-OFDMA is to use CDM in the frequency domain. This consists of spreading data symbols over all carriers forming a carrier group rather than transmitting them on separate carriers. (Previous studies which proposed to reintroduce frequency diversity in OFDM system include [8] and [9]). We will describe this technique using two simple examples, where the number of carriers per group is 2 and 4, respectively. Its generalization to higher numbers of carriers per group is straightforward

With 2 carriers per group in OFDMA and frequency-domain CDM within each carrier group, the transmitted signal for group k is

$$s_k = 1/\sqrt{2}(a_k + a_{k+N/2}) \quad (8.a)$$

$$s_{k+N/2} = 1/\sqrt{2}(a_k - a_{k+N/2}) \quad (8.b)$$

The received signal samples are

$$r_k = 1/\sqrt{2}H_k(a_k + a_{k+N/2}) + \eta_k \quad (9.a)$$

$$r_{k+N/2} = 1/\sqrt{2}H_{k+N/2}(a_k - a_{k+N/2}) + \eta_{k+N/2} \quad (9.b)$$

Note that in an OFDMA system with two carriers per group, $N/2T$ is the maximum spacing between the 2 carriers that can be achieved for all groups. If $N/2T$ is larger than the channel coherence bandwidth, then the two carriers of the same group will be affected by independent fades and CDM will significantly enhance performance.

With ZF equalization, the equalizer output samples at the for carriers k and $k + N/2$ are



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$$y_k = 1/\sqrt{2}(a_k + a_{k+N/2}) + \eta_k / H_k, \quad (10.a)$$

$$y_{k+N/2} = 1/\sqrt{2}(a_k - a_{k+N/2}) + \eta_{k+N/2} / H_{k+N/2} \quad (10.b)$$

Since each symbol is transmitted on two frequencies, the information received at these frequencies must be combined to recover the symbol at hand. For symbol a_k , this operation is

$$\tilde{a}_k = (y_k + y_{k+N/2})/\sqrt{2} = a_k + (\eta_k / H_k + \eta_{k+N/2} / H_{k+N/2})/\sqrt{2} \quad (11)$$

Assuming that the two noise samples in this expression are independent, the SNR at the detector input is

$$SNR_k = 2(\sigma_a^2 / \sigma_n^2) \frac{1}{1/|H_k|^2 + 1/|H_{k+N/2}|^2}. \quad (12)$$

If one of the two carriers is subjected to a deep fade, e.g., $|H_k| \ll |H_{k+N/2}|$, we have

$$SNR_k \approx 2|H_k|^2 (\sigma_a^2 / \sigma_n^2). \quad (13)$$

This means that the simple spreading described gives a 3 dB gain compared to the original OFDMA performance given by (7). The frequency-domain spreading matrix in the described scheme is the 2×2 Walsh-Hadamard matrix $\mathbf{W}_{2 \times 2}$ and the overall precoding matrix of dimension $N \times N$ is

$$\mathbf{P}_{N \times N} = \frac{1}{\sqrt{2}} \mathbf{W}_{2 \times 2} \otimes \mathbf{I}_{N/2}, \quad (14)$$

where \otimes denotes Kronecker product.

Proceeding similarly, we can describe SC-OFDMA with signal spreading over 4 carriers. The transmitted signal samples are in this case

$$s_k = 1/\sqrt{4}(a_k + a_{k+N/4} + a_{k+2N/4} + a_{k+3N/4}), \quad (15.a)$$

$$s_{k+N/4} = 1/\sqrt{4}(a_k - a_{k+N/4} + a_{k+2N/4} - a_{k+3N/4}) \quad (15.b)$$

$$s_{k+2N/4} = 1/\sqrt{4}(a_k + a_{k+N/4} - a_{k+2N/4} - a_{k+3N/4}), \quad (15.c)$$

$$s_{k+3N/4} = 1/\sqrt{4}(a_k - a_{k+N/4} - a_{k+2N/4} + a_{k+3N/4}) \quad (15.d)$$

This scheme uses the 4×4 Walsh-Hadamard matrix for signal spreading and

the overall precoding matrix is

$$\mathbf{P}_{N \times N} = \frac{1}{\sqrt{4}} \mathbf{W}_{4 \times 4} \otimes \mathbf{I}_{N/4}. \quad (16)$$

The received signal samples are:

$$r_k = H_k s_k + \eta_k \quad (17)$$

and the equalizer output samples are

$$y_k = s_k + \eta_k / H_k \quad (18)$$

for all k .

Symbol detection is performed by passing y_k , $y_{k+N/4}$, $y_{k+2N/4}$ and $y_{k+3N/4}$ to the inverse 4×4 Walsh-Hadamard matrix and sending the resulting signal to a threshold detector. The threshold detector input for symbol a_k is

$$\begin{aligned} \tilde{a}_k &= (y_k + y_{k+N/4} + y_{k+2N/4} + y_{k+3N/4})/\sqrt{4} \\ &= a_k + \frac{1}{\sqrt{4}} \sum_{i=0}^3 \eta_{k+iN/4} / H_{k+iN/4}. \end{aligned} \quad (19)$$

The SNR at the detector input is thus

$$SNR_k = 4(\sigma_a^2 / \sigma_n^2) \frac{1}{\sum_{i=0}^3 1/|H_{k+iN/4}|^2}. \quad (20)$$

If one of the 4 frequencies is deeply faded, i.e., $|H_k| \ll |H_{k+iN/4}|$, for $i \in \{1,2,3\}$, we have

$$SNR_k \approx 4|H_k|^2 (\sigma_a^2 / \sigma_n^2). \quad (21)$$

This expression shows that an SNR improvement of approximately 6 dB is obtained compared to (7). In the general case, the spreading matrix is $\mathbf{W}_{SF \times SF}$ and the precoding matrix can be expressed as

$$\mathbf{P}_{N \times N} = \frac{1}{\sqrt{SF}} \mathbf{W}_{SF \times SF} \otimes \mathbf{I}_{N/SF}, \quad (22)$$

where SF is the spreading factor that is equal to the number of carriers per group N/M . In the limiting case ($SF = N$), this technique coincides with MC-CDMA.

Performance Analysis

The performance of SC-OFDMA with



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frequency-domain CDM will be investigated here using three types of detectors.

ZF Equalization

The received signal vector for precoded OFDM can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{P}\mathbf{a} + \boldsymbol{\eta} \quad (23)$$

And the ZF equalizer is given by

$$\mathbf{G}_{zf} = (\mathbf{H}\mathbf{P})^\dagger \quad (24)$$

Using the precoding matrix definitions of the previous section, we can easily write the bit error probability (BER) as

$$P_b = \frac{1}{N} \sum_{k=1}^N Q\left(\sqrt{\frac{\sigma_a^2}{\sigma_\eta^2 \sum_{i=1}^N |p_{i,k}|^2 |H_i|^2}}\right) \quad (25)$$

Using this equation, the BER performance of SC-OFDMA with CDM and ZF equalization is plotted in Fig. 1 together with the simulation results shown with dots. The number of carriers is $N = 64$ and the channel transfer function is $H(f) = 1 + \rho e^{-j2\pi f T / N + \varphi}$ with $\rho = 0.9$ and $\varphi = \pi / 2$.

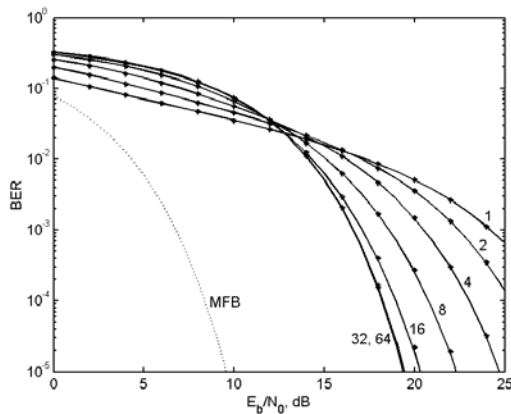


Fig. 1: Performance of OFDMA-CDM with ZF equalization ($SF = 1, 2, 4, 8, 16, 32, 64$).

The results confirm that at the BER of 10^{-5} , an SNR gain of approximately 3 dB is achieved when the size of the carrier groups and the spreading factor are doubled. This gain holds only for the first steps and it

decreases at further steps, so that in the limit (with $SF = 64$), performance of this scheme coincides with that of MC-CDMA. The figure also shows the matched filter bound (MFB), which is approximately 10 dB to the left from the MC-CDMA performance on this channel.

MMSE Equalization

Another simple receiver is the linear MMSE equalizer. This equalizer minimizes $E\|\mathbf{G}\mathbf{H}\mathbf{P}\mathbf{a} + \mathbf{G}\boldsymbol{\eta} - \mathbf{a}\|^2$ and is given by

$$\mathbf{G}_{mmse} = \mathbf{R}_a \mathbf{P}^H \mathbf{H}^H (\mathbf{H} \mathbf{P} \mathbf{R}_a \mathbf{P}^H \mathbf{H}^H + \mathbf{R}_\eta)^{-1} \quad (26)$$

For $E[\mathbf{a}\mathbf{a}^H] = \sigma_a^2 \mathbf{I}_N$, $E[\boldsymbol{\eta}\boldsymbol{\eta}^H] = \sigma_\eta^2 \mathbf{I}_N$ and a unitary precoding matrix \mathbf{P} , it reduces to

$$\mathbf{G}_{mmse} = \mathbf{P}^H \text{diag}\left(\frac{|H_1|^2}{|H_1|^2 + \sigma_\eta^2 / \sigma_a^2}, \dots, \frac{|H_N|^2}{|H_N|^2 + \sigma_\eta^2 / \sigma_a^2}\right) \quad (27)$$

The BER performance has been studied in [10] making a Gaussian approximation for the interference. For QPSK, it is expressed as

$$P_b = \frac{1}{N} \sum_{k=1}^N Q\left(\sqrt{\frac{\gamma_k}{1 - \gamma_k}}\right) \quad (28)$$

where γ_k is the desired signal component at the output of the MMSE equalizer, given by

$$\gamma_k = \sum_{i=1}^N \frac{|p_{i,k}|^2 |H_i|^2}{|H_i|^2 + \sigma_\eta^2 / \sigma_a^2} \quad (29)$$

Fig. 2 depicts the theoretical performance (computed using (28)) and the simulated performance of SC-OFDMA/CDM with MMSE equalization. As in the ZF case, we notice that performance is improved as the spreading factor is increased. Again, the SNR increase becomes smaller from step to step and performance converges to that of MC-CDMA as SF approaches N . But contrary to ZF equalization, this technique improves performance even at low SNR values.

ML Detection

The optimum receiver for OFDMA-CDM is the ML detector, which selects the sequence



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with the smallest Euclidean distance to the received signal. It can be written as

$$\hat{\mathbf{a}} = \underset{\mathbf{a} \in A}{\operatorname{argmin}} \|\mathbf{r} - \mathbf{H}\mathbf{P}\mathbf{a}\|^2, \quad (30)$$

where A is the set of all possible input symbol vectors. The ML detector complexity increases exponentially with the spreading factor and it is not practical for $SF > 8$. In Fig. 3, performance of the ML detector is given for $SF = 2, 4$ and 8. These results indicate that ML detection performance is close to the MFB even with $SF = 8$.

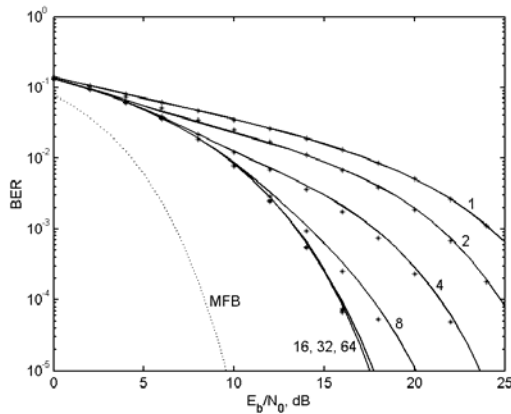


Fig. 2: Performance of OFDMA-CDM with MMSE equalization ($SF = 1, 2, 4, 8, 16, 32, 64$).

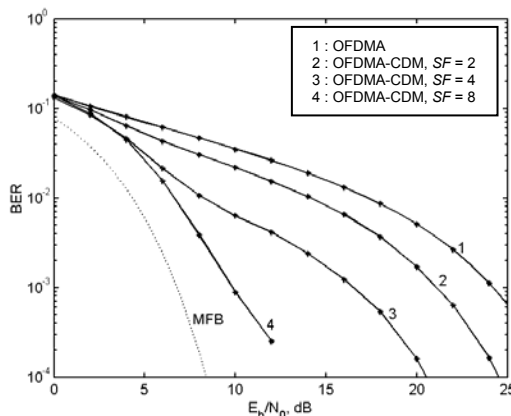


Fig. 3: Performance of OFDMA-CDM with maximum likelihood detection ($SF = 2, 4, 8$).

Conclusions

We have described and analyzed the performance of an OFDMA scheme in which

the carriers assigned to a user are uniformly distributed across the channel bandwidth and frequency-domain CDM is used. Its performance was investigated using ZF and MMSE equalization as well as ML detection, and the results have shown that this multiple access technique achieves a BER performance very close to that of MC-CDMA, while increasing the cell range several folds as compared to TDMA and MC-CDMA. As a conclusion, the presented scheme recovers most of the frequency-diversity loss that is inherent to OFDM-based systems.

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