Recent Results in Space-Time Code Design

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***Abstract*— Multiple-input multiple-output (MIMO) techniques have become an essential component of broadband wireless communications systems. For example, the recently developed IEEE 802.16e specifications include three 2×2 MIMO profiles, and two of these profiles are mandatory on the downlink of Mobile WiMAX systems. One of these MIMO profiles is full rate, and the other offers full diversity, but neither of them has both of the desired features. The third profile, namely, Matrix C, which is not mandatory, is both a full rate and a full diversity space-time code (STC), but it has a high decoder complexity. Recently, the attention was turned to the decoder complexity issue and including this in the design criteria, new full-rate and high-rate STCs were proposed as alternatives to Matrix C. In this paper, we review conventional STCs and describe these recent designs, which lead to an optimum decoder of reduced complexity.**

***Index Terms*— MIMO systems; ML detection; space-time codes; WiMAX; LTE**

I. Introduction

Multiple-input multiple-output (MIMO) techniques which consist of using multiple antennas at both transmitter and receiver can provide spatial diversity, multiplexing gain, interference suppression, and make various tradeoffs between them. These techniques have been incorporated in all of the recently developed wireless communications system specifications including the IEEE 802.11n standard for local area networks and the IEEE 802.16e-2005 standard [1] for mobile broadband wireless access systems. Mobile WiMAX systems are based on the scalable OFDMA mode of IEEE 802.16e-2005 specifications and use a subset of the different options included in them.

Regarding MIMO options, the WiMAX Forum has specified two mandatory profiles for use on the downlink. One of them is based on the space–time code (STC) proposed by Alamouti for transmit diversity [2]. This code provides perfect second-order diversity when used with a single receive antenna and fourth-order diversity when used with two antennas at the receiver. But it is only half-rate, because it only transmits two symbols using two time slots and two transmit antennas. The other profile is spatial multiplexing (SM), which uses two transmit antennas to transmit two independent data streams. This scheme is full-rate, but it does not benefit from any diversity gain at the transmitter, and at best, it provides second-order diversity with two antennas at the receiver [3].

For future evolutions of the WiMAX standard as well for future releases of 3GPP LTE, it is highly desirable to include a new code combining the respective advantages of the Alamouti code and the SM while avoiding their drawbacks. Such a code actually exists in the IEEE 802.16e-2005 specifications, where it is referred to as Matrix C. The matrix C is a variant of the Golden code [4], which is known to be one of the best STCs of size 2×2. However, this code has a tremendous decoding complexity which grows with the fourth power of the signal constellation size, and this makes it impractical for low-cost wireless systems.

Recently, the present authors developed a full-rate and full-diversity 2×2 STC design, whose optimum detection complexity grows only quadratically with the size of the signal constellation [5]. Compared to Matrix C (and to the Golden code), this approach reduces the optimum decoder complexity by a factor of 256 (resp. 4,096) for the 16-QAM (resp. 64-QAM) signal constellation. Using a similar approach, we also developed a rate-3/4 code, whose optimum detection complexity grows only linearly with the size of the signal constellation [6].

In this paper, we will describe these new STC designs and report performance results. First, in the next section, we briefly discuss the general design criteria for STCs. Next, in Section 3, we recall the three 2×2 STCs which appear in the IEEE 802.16e specifications. In Section 4, we describe our full-rate and full-diversity 2×2 STC with quadratic optimum decoder complexity. Then, in Section 5, we describe a rate-3/4 variant of this technique with linear optimum decoder complexity. Finally, we present some performance results in Section 6, and give our conclusions in Section 7.

II. STC Design Criteria

We start with a brief summary of the most common design criteria used for STC design when the transmitter does not have any channel state information. For 2×2 MIMO systems, we write

$$\mathbf{R} = \mathbf{H}\mathbf{X} + \mathbf{Z}, \qquad (1)$$

where $\mathbf{H}$ is the 2×2 channel matrix whose elements are the complex channel gains, and $\mathbf{X}$ is the 2×2 codeword matrix

$$\mathbf{X} = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix}. \qquad (2)$$

In (1), $\mathbf{R}$ includes the received signals, and $\mathbf{Z}$ denotes the matrix of additive noise samples. Code optimization generally relies on the analysis of pairwise error probability (PEP) $P(\mathbf{X} \to \hat{\mathbf{X}})$ which is the probability that $\hat{\mathbf{X}}$ is detected while $\mathbf{X}$ is transmitted. The goal is to minimize the error probability, but the analysis of exact error performance is difficult, and therefore one resorts to the union bound

$$P_s \le \sum_{\mathbf{X}\in X} \sum_{\hat{\mathbf{X}}\neq \mathbf{X}} P(\mathbf{X} \to \hat{\mathbf{X}}), \qquad (3)$$

where X denotes the codebook of the space-time code. A Chernoff bound analysis of the PEP was performed in [7], and this analysis led to the following design criteria:

1) Rank Criterion: In order to achieve the maximum diversity, the diversity gain

$$d(X) = \min_{\substack{\mathbf{X},\hat{\mathbf{X}}\in X \\ \mathbf{X}\neq\hat{\mathbf{X}}}} \text{rank}(\Delta\Delta^H), \quad (4)$$

where $\Delta = \mathbf{X} - \hat{\mathbf{X}}$, should be maximized. If $\Delta\Delta^H$ is a full rank matrix for all codeword pairs, then the code is said to have *full diversity*.

2) Determinant Criterion: The coding gain is defined as

$$\delta = \min_{\substack{\mathbf{X},\hat{\mathbf{X}}\in X \\ \mathbf{X}\neq\hat{\mathbf{X}}}} \prod_{l=1}^{d(X)} \lambda_l(\Delta\Delta^H), \quad (5)$$

where $\lambda_l(\cdot)$ denotes the lth largest eigenvalue of the enclosed matrix. In order to obtain the best performance, the coding gain should be maximized for a given average transmit power.

It should be noted that, for high signal-to-noise ratio (SNR) values, the most important parameter is the diversity gain which dominates the steepness of the bit-error rate (BER) curve. Afterwards, it is the coding gain which should be maximized.

III. CONVENTIONAL MIMO PROFILES

In this section, we briefly recall the three 2×2 STCs used in the IEEE 802.16e-2005 specifications, two of which are mandatory profiles on the downlink of mobile WiMAX.

A. Alamouti's Transmit Diversity

The first MIMO profile is the simple STC scheme which was introduced by Alamouti [2] for transmit diversity. In the IEEE 802.16e-2005 specifications, this scheme is referred to as Matrix A. Originally, Alamouti's STC was proposed to avoid the use of receive diversity and keep the subscriber stations simple. In OFDMA-based mobile WiMAX systems, this technique is applied subcarrier by subcarrier and can be described as follows:

Suppose that (s_1, s_2) represents a group of two consecutive symbols in the input data stream to be transmitted. During the first symbol period t_1, transmit (Tx) antenna 1 transmits symbol s_1 and Tx antenna 2 transmits symbol s_2. Next, during the second symbol period t_2, Tx antenna 1 transmits symbol s_2^* and Tx antenna 2 transmits symbol $-s_1^*$. Denoting the channel response from Tx1 to the receiver (Rx) by h_1 and the channel response from Tx2 to the Rx by h_2, the received signal samples corresponding to the symbol periods t_1 and t_2 can be written as:

$$r_1 = h_1 s_1 + h_2 s_2 + n_1, \quad (6.a)$$

$$r_2 = h_1 s_2^* - h_2 s_1^* + n_2, \quad (6.b)$$

where n_1 and n_2 are additive noise terms.

The receiver computes the following signals to estimate the symbols s_1 and s_2:

$$x_1 = h_1^* r_1 - h_2 r_2^* = \left(|h_1|^2 + |h_2|^2\right)s_1 + h_1^* n_1 - h_2 n_2^* \quad (7.a)$$

$$x_2 = h_2^* r_1 + h_1 r_2^* = \left(|h_1|^2 + |h_2|^2\right)s_2 + h_2^* n_1 + h_1 n_2^* \quad (7.b)$$

These expressions clearly show that x_1 (resp. x_2) can be sent to a threshold detector to estimate symbol s_1 (resp. symbol s_2) without interference from the other symbol. Moreover, since the useful signal coefficient is the sum of the squared moduli of two independent fading channels, these estimations benefit from perfect second-order diversity, which is equivalent to Rx diversity under maximum-ratio combining (MRC).

Alamouti's transmit diversity can also be combined with MRC when 2 antennas are used at the receiver. In this scheme, the received signal samples corresponding to the symbol periods t_1 and t_2 can be written as:

$$r_{11} = h_{11}s_1 + h_{12}s_2 + n_{11} \quad (8.a)$$

$$r_{12} = h_{11}s_2^* - h_{12}s_1^* + n_{12} \quad (8.b)$$

for the first receive antenna, and

$$r_{21} = h_{21}s_1 + h_{22}s_2 + n_{21} \quad (9.a)$$

$$r_{22} = h_{21}s_2^* - h_{22}s_1^* + n_{22} \quad (9.b)$$

for the second receive antenna. In these expressions, h_{ji} designates the channel response from Tx i to Rx j, with i, j = 1, 2, and n_{ji} designates the noise on the corresponding channel. This MIMO scheme does not give any spatial multiplexing gain, but it has 4th-order diversity, which can be fully recovered by a simple receiver. Indeed, the optimum receiver estimates the transmitted symbols s_1 and s_2 using:

$$\begin{aligned} x_1 &= h_{11}^* r_{11} - h_{12} r_{12}^* + h_{21}^* r_{21} - h_{22} r_{22}^* \\ &= \left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2\right)s_1 + \eta_1 \end{aligned} \quad (10.a)$$

and

$$\begin{aligned} x_2 &= h_{12}^* r_{11} + h_{11} r_{12}^* + h_{22}^* r_{21} + h_{21} r_{22}^* \\ &= \left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2\right)s_2 + \eta_2 \end{aligned} \quad (10.b)$$

with $\eta_1 = h_{11}^* n_{11} - h_{12} n_{12}^* + h_{21}^* n_{21} - h_{22} n_{22}^*$

and $\eta_2 = h_{12}^* n_{11} + h_{11} n_{12}^* + h_{22}^* n_{21} + h_{21} n_{22}^*$.

These equations give evidence that the receiver indeed leads to fourth-order diversity, which is the maximum spatial diversity of a 2×2 MIMO system.

B. Spatial Multiplexing

The second MIMO profile included in mobile WiMAX systems is the 2×2 MIMO technique based on the so-called matrix $\mathbf{B}$ = (s_1, s_2)^T. This system performs pure spatial multiplexing and does not offer any diversity gain from the Tx side. But it does offer a diversity gain of 2 on the receiver side when detected using maximum-likelihood (ML) detection [3].

To describe the 2×2 spatial multiplexing, we omit the time and frequency dimensions, leaving only the space dimension. The symbols transmitted by Tx1 and Tx2 in parallel are denoted as s_1 and s_2, respectively. Denoting by h_{ji} the channel response from Tx i to Rx j (i, j = 1, 2), the signals received by the two Rx antennas are given by

$$r_1 = h_{11}s_1 + h_{12}s_2 + n_1 , \tag{11.a}$$

$$r_2 = h_{21}s_1 + h_{22}s_2 + n_2 , \tag{11.b}$$

which can be written in matrix form as:

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}. \tag{12}$$

The ML detector makes an exhaustive search over all possible values of the transmitted symbols and decides in favor of (s_1, s_2) which minimizes the Euclidean distance:

$$D(s_1, s_2) = \left\{ |r_1 - h_{11}s_1 - h_{12}s_2|^2 + |r_2 - h_{21}s_1 - h_{22}s_2|^2 \right\}. \tag{13}$$

The complexity of the ML detector grows quadratically with the size of the signal constellation, and this motivates the use of simpler suboptimum detectors in practice.

C. Matrix C

This code matrix was included in the IEEE 802.16e–2005 specifications in order to provide full-rate, full-diversity and a high coding gain. For a group of 4 symbols (s_1, s_2, s_3, s_4), the matrix is given by

$$\mathbf{X}_C = \frac{1}{\sqrt{1 + r^2}} \begin{bmatrix} s_1 + jrs_4 & rs_2 + s_3 \\ s_2 - rs_3 & jrs_1 + s_4 \end{bmatrix}, \tag{14}$$

where $r = (-1 + \sqrt{5})/2$ and $j = \sqrt{-1}$.

With 2 receive antennas, this code matrix leads to 4th-order spatial diversity and it achieves substantially better performance than the SM code (Matrix B). In fact, $\mathbf{X}_C$ results in the same bit error probability as the Golden code, which is the best-known 2×2 STC. Compared to the Golden code, construction of $\mathbf{X}_C$ requires a smaller number of multiplications, but, as indicated earlier, both the Golden code and Matrix C have a large detection complexity.

The optimum receiver evaluates the ML function for all symbol quadruplets (s_1, s_2, s_3, s_4) and selects the one which maximizes this function. The ML function evaluated for (s_1, s_2, s_3, s_4) is actually the squared Euclidean distance between the received noisy signal and the noiseless signal corresponding to that quadruplet, and can be expressed as the squared Frobenius norm

$$D(s_1, s_2, s_3, s_4) = \|\mathbf{Y} - \mathbf{HX}\|^2 . \tag{15}$$

For a signal constellation with M points, this receiver involves the computation of M^4 Euclidean distances and selects the symbol quadruplet minimizing this distance. The optimum receiver complexity is, therefore, proportional to 16^4 = 65,536 for a 16-QAM signal constellation, and to 64^4 = 16,777,216 for a 64-QAM signal constellation. Of course, this is prohibitive in practical applications. Therefore, one resorts to suboptimum receivers which may degrade the performance severely. One possible solution is to use sphere detection (SD) [8] whose performance and complexity are upper bounded by those of ML detection based on exhaustive search. The major issue in the implementation of SD is the choice of the initial radius and of the order in which the symbols are examined. This can dramatically improve or degrade the complexity of SD.

IV. FULL-RATE 2X2 STC

Now, we describe our recently proposed full-rate 2×2 STC design which maximizes both the diversity gain and the coding gain, while leading to an optimum detector of reduced complexity [5]. More specifically, this STC is a full-rate, full-diversity 2×2 STC whose optimum receiver has a complexity that is *only* proportional to M^2, where M is the size of the signal constellation. Thus, the number of Euclidean distance computations in the optimum detector is reduced to 16^2 = 256 for the 16-QAM signal constellation and to 64^2 = 4,096 for the 64-QAM signal constellation. Comparing these numbers to those associated to the Golden code (or Matrix C), it becomes clear that this code makes the implementation of full-rate, full-diversity 2×2 STCs with optimum detection very realistic. We now describe this newly proposed code.

A group of 4 data symbols (s_1, s_2, s_3, s_4) in the proposed code design is transmitted as follows:

$$\mathbf{X} = \begin{bmatrix} as_1 + bs_3 & -cs_2^* - ds_4^* \\ as_2 + bs_4 & cs_1^* + ds_3^* \end{bmatrix}, \tag{16}$$

where a, b, c, and d are complex-valued design parameters and the star designates complex conjugate.

In this matrix representation, the first column represents the symbol combinations transmitted during a first symbol interval t_1 and the second column represents the symbol combinations transmitted during a second symbol interval t_2. The first row of the matrix gives the symbol combinations transmitted from the first Tx antenna, and second row of the matrix gives the symbol combinations transmitted from the second Tx antenna. In other words, $as_1 + bs_3$ is transmitted from Tx antenna 1 during the first symbol interval t_1, $as_2 + bs_4$ is transmitted from Tx antenna 2 during the first symbol interval t_1, $-cs_2^* - ds_4^*$ is transmitted from Tx antenna 1 during the second symbol interval t_2, and $cs_1^* + ds_3^*$ is transmitted from Tx antenna 2 during the second symbol interval t_2.

On the first receive antenna, the two signals received at the first and second symbol intervals are:

$$r_1 = h_{11}(as_1 + bs_3) + h_{12}(as_2 + bs_4) + n_1 \tag{17.a}$$
$$r_2 = h_{11}(-cs_2^* - ds_4^*) + h_{12}(cs_1^* + ds_3^*) + n_2 . \tag{17.b}$$

Similarly, we have on the second Rx antenna:

$$r_3 = h_{21}(as_1 + bs_3) + h_{22}(as_2 + bs_4) + n_3 \tag{18.a}$$
$$r_4 = h_{21}(-cs_2^* - ds_4^*) + h_{22}(cs_1^* + ds_3^*) + n_4 \tag{18.b}$$

where n_i , for $i = 1, \ldots, 4$, are the additive noise terms.

The ML detector makes an exhaustive search over all possible values of the transmitted symbols and decides in favor of the quadruplet (s_1, s_2, s_3, s_4) which minimizes the Euclidean distance:

$$\begin{aligned} D(s_1,s_2,s_3,s_4) = \Big\{ & \left| r_1 - h_{11}(as_1 + bs_3) - h_{12}(as_2 + bs_4) \right|^2 \\ & + \left| r_2 - h_{11}\left(-cs_2^* - ds_4^*\right) - h_{12}\left(cs_1^* + ds_3^*\right) \right|^2 \\ & + \left| r_3 - h_{21}(as_1 + bs_3) - h_{22}(as_2 + bs_4) \right|^2 \\ & + \left| r_4 - h_{21}\left(-cs_2^* - ds_4^*\right) - h_{22}\left(cs_1^* + ds_3^*\right) \right|^2 \Big\} \end{aligned}$$

As in Matrix C, an exhaustive search involves the computation of M^4 metrics and M^4-1 comparisons. But the proposed STC design lends itself to a low-complexity implementation of the ML detector as we now show.

From the received signal samples (r_1, r_2, r_3, r_4), let us compute the following signals:

$$w_1 = r_1 - b(h_{11}s_3 + h_{12}s_4) = a(h_{11}s_1 + h_{12}s_2) + n_1 \quad (19.a)$$
$$w_2 = r_2 - d\left(h_{12}s_3^* - h_{11}s_4^*\right) = c\left(h_{12}s_1^* - h_{11}s_2^*\right) + n_2 \quad (19.b)$$
$$w_3 = r_3 - b(h_{21}s_3 + h_{22}s_4) = a(h_{21}s_1 + h_{22}s_2) + n_3 \quad (19.c)$$
$$w_4 = r_4 - d\left(h_{22}s_3^* - h_{21}s_4^*\right) = c\left(h_{22}s_1^* - h_{21}s_2^*\right) + n_4 \,. \quad (19.d)$$

Next, from (w_1, w_2, w_3, w_4), we compute:

$$h_{11}^* w_1 = a\left(|h_{11}|^2 s_1 + h_{11}^* h_{12} s_2\right) + h_{11}^* n_1 \quad (20.a)$$
$$h_{12} w_2^* = c^*\left(|h_{12}|^2 s_1 - h_{11}^* h_{12} s_2\right) + h_{12} n_2^* \quad (20.b)$$
$$h_{21}^* w_3 = a\left(|h_{21}|^2 s_1 + h_{21}^* h_{22} s_2\right) + h_{21}^* n_3 \quad (20.c)$$
$$h_{22} w_4^* = c^*\left(|h_{22}|^2 s_1 - h_{21}^* h_{22} s_2\right) + h_{22} n_4^* \,. \quad (20.d)$$

From those signals, we next compute the signal y_1 given by:

$$\begin{aligned} y_1 &= \left(h_{11}^* w_1 + h_{21}^* w_3\right)/a + \left(h_{12} w_2^* + h_{22} w_4^*\right)/c^* \\ &= \left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2\right) s_1 + \eta_1 \end{aligned} \quad (21)$$

with $\eta_1 = \left(h_{11}^* n_1 + h_{21}^* n_3\right)/a + \left(h_{12} n_2^* + h_{22} n_4^*\right)/c^*$.

It can be seen that the signal y_1 has no terms involving symbol s_2, and the coefficient of the term in s_1 indicates that estimation of s_1 benefits from 4th-order spatial diversity. By sending this signal to a threshold detector, we get the ML estimate of symbol s_1 conditional on (s_3, s_4). Note that the elimination of the terms involving s_2 is possible if and only if the respective coefficients of the symbols s_1 and s_2 in each column of the code matrix are identical.

Similarly, we compute the intermediate signals $h_{12}^* w_1$, $h_{11} w_2^*$, $h_{22}^* w_3$, $h_{21} w_4^*$, and then,

$$\begin{aligned} y_2 &= \left(h_{12}^* w_1 + h_{22}^* w_3\right)/a - \left(h_{11} w_2^* + h_{21} w_4^*\right)/c^* \\ &= \left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2\right) s_2 + \eta_2 \end{aligned} \quad (22)$$

with $\eta_2 = \left(h_{12}^* n_1 + h_{22}^* n_3\right)/a - \left(h_{11} n_2^* + h_{21} n_4^*\right)/c^*$.

As previously, signal y_2 has no terms involving symbol s_1 and the coefficient of the term in s_2 shows that estimation of s_2 benefits from 4th-order spatial diversity. By sending y_2 to a threshold detector, we get the ML estimate of symbol s_2 conditional on (s_3, s_4). ML estimation of s_1 and s_2 conditional on (s_3, s_4) is illustrated in Figure 1. In this way, for a given symbol pair (s_3, s_4), we get the ML estimate of (s_1, s_2), which we denote (s_1^{ML}, s_2^{ML}). Now, instead of computing the metric $D(s_1, s_2, s_3, s_4)$ for all (s_1, s_2, s_3, s_4) values, we only need to compute it for $(s_1^{ML}, s_2^{ML}, s_3, s_4)$, with s_3 and s_4 spanning the signal constellation. Specifically, let (s_3^k, s_4^l) indicate that symbol s_3 takes the kth point of the signal constellation and symbol s_4 takes the lth point of the signal constellation. The optimum receiver computes the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^{ML}, s_2^{ML}, s_3^k, s_4^l)$, where k, l = 1, 2, …, M. This procedure, which is illustrated in Figure 2, reduces the ML receiver complexity from M^4 to M^2.

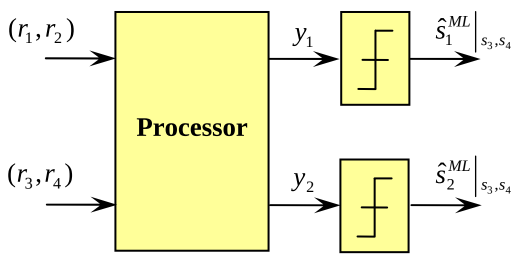

Figure 1. Processing of the received signals to determine the ML estimate of symbols s_1 and s_2 conditional on a particular combination of symbols s_3 and s_4.

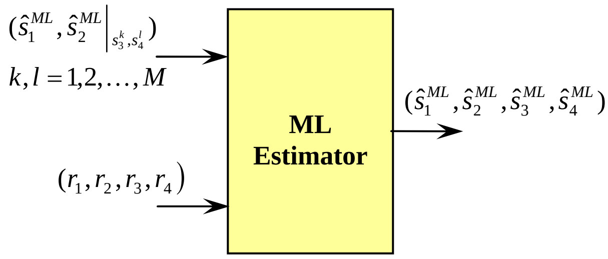

Figure 2. Second stage of the estimator.

Note that the special structure of (16) allows the ML detector also to work the other way round: Instead of deriving the ML estimate of (s_1, s_2) conditional on (s_3^k, s_4^l) and then computing the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^{ML}, s_2^{ML}, s_3^k, s_4^l)$, we can first estimate (s_3, s_4) conditional on (s_1^k, s_2^l), then compute the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^k, s_2^l, s_3^{ML}, s_4^{ML})$, k, l = 1, 2, …., M, and select the quadruplet (s_1, s_2, s_3, s_4) minimizing the metric.

It is instructive to point out here that the described detector is optimum only when the magnitudes of a and c

(alternatively the magnitudes of b and d for the reverse detection order) are equal. This can be easily seen by looking at the SNR at the receiver input and then at the threshold detector input. Indeed, these two SNR values are the same if and only if $|a|=|c|$ for forward detection and $|b|=|d|$ for reverse detection.

The a, b, c, d parameters in the code matrix are design parameters to be optimized in order to obtain a full-diversity STC with a large coding gain. However, this task is infeasible especially for higher constellation sizes. Fortunately, the transmit power constraints can further decrease the number of parameters to be optimized.

In terms of the transmitted power, the desired conditions can be expressed as

$$|a|^2+|b|^2=1=|c|^2+|d|^2$$
$$|a|^2+|c|^2=1=|b|^2+|d|^2 .$$

The first condition ensures an equal transmit power at each symbol time, while the second condition ensures that equal total power is transmitted for each symbol. These equalities together with the constraint $|a|=|c|$ for optimal detection lead immediately to the fact that all the design parameters should have the same magnitude, *i.e.*,

$$|a|=|b|=|c|=|d|=1/\sqrt{2} .$$

Without any loss of generality, we take $a=c=1/\sqrt{2}$ (since they do not affect the coding gain) and make an exhaustive search to optimize the parameters b and d.

V. DERIVATION OF A RATE-3/4 STC

We now describe a simple modification of the STC given by (16) which further reduces the ML decoder complexity [6]. This consists of setting $s_4=s_3$ in the coding matrix and scaling the power of this symbol. This leads to the following 2×2 code of rate 3/4:

$$\mathbf{X}_{3/4}=\begin{bmatrix} as_1+bs_3/\sqrt{2} & -(cs_2^*+ds_3^*/\sqrt{2}) \\ as_2+bs_3/\sqrt{2} & cs_1^*+ds_3^*/\sqrt{2}\end{bmatrix}. \qquad (23)$$

For the sake of simplicity, we only consider the 2 Rx antenna case to describe the detector. The results can be easily extended to any number of Rx antennas.

On the first Rx antenna, let the two signals received at the first and second symbol intervals be

$$r_1=h_{11}\left(as_1+bs_3/\sqrt{2}\right)+h_{12}\left(as_2+bs_3/\sqrt{2}\right)+n_1 \quad (24.a)$$
$$r_2=h_{11}\left(-cs_2^*-ds_3^*/\sqrt{2}\right)+h_{12}\left(cs_1^*+ds_3^*/\sqrt{2}\right)+n_2 . \quad (24.b)$$

Similarly, on the second Rx antenna we have

$$r_3=h_{21}\left(as_1+bs_3/\sqrt{2}\right)+h_{22}\left(as_2+bs_3/\sqrt{2}\right)+n_3 \quad (25.a)$$
$$r_4=h_{21}\left(-cs_2^*-ds_3^*/\sqrt{2}\right)+h_{22}\left(cs_1^*+ds_3^*/\sqrt{2}\right)+n_4 \quad (25.b)$$

where the n_i's are circularly symmetric additive Gaussian noise terms with spectral density N_0.

The ML detector makes an exhaustive search over all possible values of the transmitted symbols and decides in favor of the symbol triplet (s_1,s_2,s_3), which minimizes the Euclidean distance denoted by $D(s_1,s_2,s_3)$. The exhaustive search involves the computation of M^3 metrics and M^3-1 comparisons. But following the same steps as in the previous section, we get an ML detector whose complexity grows only linearly with the constellation size.

From the received signal samples (r_1,r_2,r_3,r_4), let us compute the following intermediate signals:

$$z_1=r_1-b(h_{11}+h_{12})s_3/\sqrt{2} \quad (26.a)$$
$$z_2=r_2-d(h_{11}+h_{12})s_3^*/\sqrt{2} \quad (26.b)$$
$$z_3=r_3-b(h_{21}+h_{22})s_3/\sqrt{2} \quad (26.c)$$
$$z_4=r_4-d(h_{21}+h_{22})s_3^*/\sqrt{2} \quad (26.d)$$

for a given value of the symbol s_3. Then, from (24.a) – (25.b), we have

$$z_1=a\left(h_{11}s_1+h_{12}s_2\right)+n_1 \quad (27.a)$$
$$z_2=c\left(h_{12}s_1^*-h_{11}s_2^*\right)+n_2 \quad (27.b)$$
$$z_3=a\left(h_{21}s_1+h_{22}s_2\right)+n_3 \quad (27.c)$$
$$z_4=c\left(h_{22}s_1^*-h_{21}s_2^*\right)+n_4 \quad (27.d)$$

From these intermediate signals, we next compute:

$$h_{11}^*z_1=a\left(|h_{11}|^2s_1+h_{11}^*h_{12}s_2\right)+h_{11}^*n_1 \quad (28.a)$$
$$h_{12}z_2^*=c^*\left(|h_{12}|^2s_1-h_{11}^*h_{12}s_2\right)+h_{12}n_2^* \quad (28.b)$$
$$h_{21}^*z_3=a\left(|h_{21}|^2s_1+h_{21}^*h_{22}s_2\right)+h_{21}^*n_3 \quad (28.c)$$
$$h_{22}z_4^*=c^*\left(|h_{22}|^2s_1-h_{21}^*h_{22}s_2\right)+h_{22}n_4^* . \quad (28.d)$$

Finally, we compute the signal y_1 as:

$$\begin{aligned} y_1&=\left(h_{11}^*z_1+h_{21}^*z_3\right)/a+\left(h_{12}z_2^*+h_{22}z_4^*\right)/c^* \\ &=\left(|h_{11}|^2+|h_{12}|^2+|h_{21}|^2+|h_{22}|^2\right)s_1+w_1 \end{aligned} \quad (29)$$

with $w_1=\left(h_{11}^*n_1+h_{21}^*n_3\right)/a+\left(h_{12}n_2^*+h_{22}n_4^*\right)/c^*$.

Notice that the signal y_1 has no terms involving symbol s_2 and the coefficient of the term in s_1 indicates that estimation of s_1 benefits from full fourth-order diversity. By sending this signal to a threshold detector, we get the ML estimate of symbol s_1 conditional on s_3. Note that, similar to the full-rate case, the elimination of the terms involving s_2 can be possible if and only if the coefficients of the symbols s_1 and s_2 are identical in each column of the code matrix.

Similar to (28.a) – (28.d), we also compute $h_{12}^*z_1$, $h_{11}z_2^*$, $h_{22}^*z_3$, $h_{21}z_4^*$, and then, we obtain

$$\begin{aligned} y_2&=\left(h_{12}^*z_1+h_{22}^*z_3\right)/a-\left(h_{11}z_2^*+h_{21}z_4^*\right)/c^* \\ &=\left(|h_{11}|^2+|h_{12}|^2+|h_{21}|^2+|h_{22}|^2\right)s_2+w_2 \end{aligned} \quad (30)$$

with $w_2 = \left(h_{12}^* n_1 + h_{22}^* n_3\right)/a - \left(h_{11} n_2^* + h_{21} n_4^*\right)/c^*$.

As previously, signal y_2 has no terms involving symbol s_1, and estimation of s_2 benefits from fourth-order diversity. By sending y_2 to a threshold detector, we get the ML estimate of symbol s_2 conditional on s_3.

In this way, for a given value of symbol s_3, we get the ML estimate of (s_1, s_2), which we denote by (s_1^{ML}, s_2^{ML}). Instead of computing the ML metric $D(s_1, s_2, s_3)$ for all possible (s_1, s_2, s_3) values, we only need to compute it for $(s_1^{ML}, s_2^{ML}, s_3)$. In other words, the optimum Rx computes the metric $D(s_1, s_2, s_3)$ for $(s_1^{ML}, s_2^{ML}, s_3^k)$, where k = 1, 2, …, M, and this reduces the ML receiver complexity from M^3 to M.

VI. PERFORMANCE RESULTS

We now provide some performance comparisons between the full-rate 2×2 matrices defined in the IEEE 802.16e-2005 standard and the 2×2 STC described in Section IV. To set the values of the b and d parameters in (16), we have performed an exhaustive search so as to maximize the coding gain (and thus to ensure full diversity) for QPSK signaling. This optimization led to the parameter values $b = \left[(1-\sqrt{7}) + i(1+\sqrt{7})\right]/(4\sqrt{2})$ and $d = e^{-i\pi/2} b$, which lead to a coding gain of 2 for all of the presented constellations.

Figure 3 shows the BER performance comparisons between the proposed scheme, and Matrix B (SM) and Matrix C of IEEE 802.16e-2005 specifications using an uncorrelated Rayleigh fading channel as a function of E_b/N_0, E_b denoting the average signal energy per bit and N_0 denoting the noise spectral density.

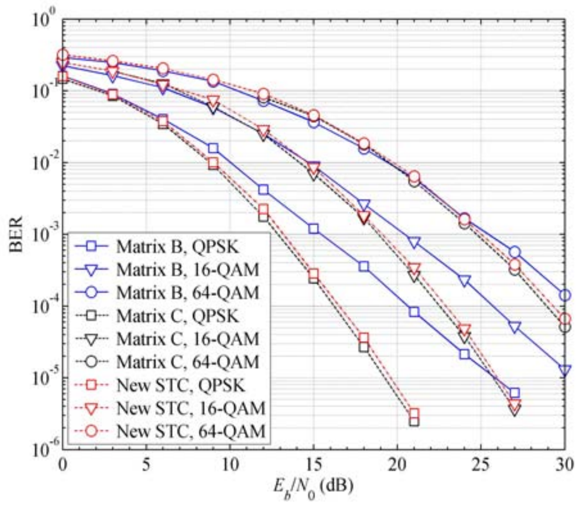

Figure 3. Performance comparison of the new 2×2 STC with Matrix B and Matrix C.

The results corresponding to Matrix B and the proposed scheme are obtained using the optimum detection for all constellation sizes. Those corresponding to Matrix C are obtained with ML detection for QPSK and sphere decoding [8] for the higher constellations. It can be seen that the slopes of the BER curves are much higher for Matrix C and the proposed scheme compared to Matrix B. As a result, these schemes substantially outperform Matrix B at high SNR values. More importantly, the proposed scheme gives essentially the same results as Matrix C at substantially lower complexity.

We next provide a performance comparison between the rate-3/4 STC presented in Section V and the two MIMO profiles included in the mobile WiMAX specifications, namely Alamouti's STC (Matrix A) and the SM (Matrix B). Two receive antennas were used in all of these schemes. The simulations were carried out for the QPSK, 16-QAM and 64-QAM signal constellations, and no channel coding was used. Fig. 4 shows the BER performance as a function of E_s/N_0 (E_s = average transmitted signal energy per antenna use) on an uncorrelated Rayleigh fading channel. The b and d parameters in our code were set as $b = d = (1 + j\sqrt{7})/4$. With these values, the proposed STC maximizes the diversity gain and, therefore, it achieves the same BER curve slope as Alamouti's STC with a coding gain, which is independent of the constellation size. This is a crucial property which is satisfied by the best known codes since we do not want vanishing determinants.

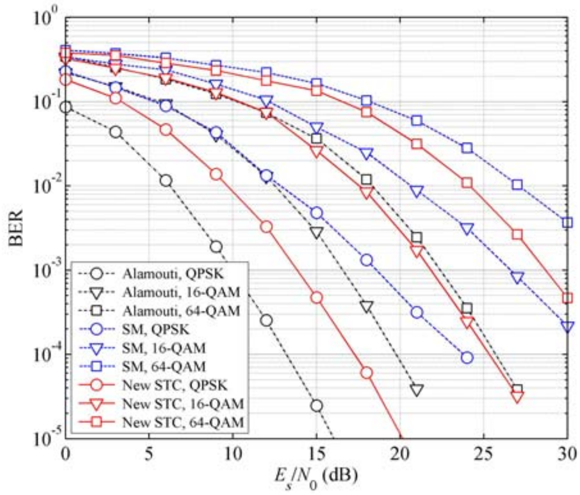

Figure 4. Performance comparison of the rate-3/4 STC with Matrix A and Matrix B.

The results of Fig. 4 indicate that Matrix A achieves a BER of 10^{-3} with an SNR of 10 dB for QPSK, 16.6 dB for 16-QAM, and 22.4 dB for 64-QAM. Next, we can observe that Matrix B achieves this BER with an SNR of 18.6 dB for QPSK and 26.6 dB for 16-QAM. With 64-QAM, this MIMO scheme requires an SNR well in excess of 30 dB to reach this performance level. Finally, our new STC achieves a BER of 10^{-3} with an SNR of 13.8 dB for QPSK, 21.7 dB for 16-QAM, and 28.6 dB for 64-QAM.

Clearly, Matrix A has the best BER performance among these schemes, but also the lowest bit rate on a given channel bandwidth. The SM scheme doubles the bit rate, but it involves a strong SNR loss, which increases at lower BER values. (This is due to the lower BER curve slope.) As evidenced from these results, the proposed rate-3/4 scheme is an interesting alternative to these two MIMO schemes, as it substantially improves BER performance compared to Matrix B at the price of a 25% decrease in bit rate, and it increases the bit rate by 50% compared to Matrix A at the price of some SNR loss.

A closer examination of the results shows that at the spectral efficiency of 3 bits per antenna use, the proposed technique outperforms Matrix A. Indeed, the new STC with 16-QAM and Matrix A with 64-QAM have the spectral efficiency of 3 bits per antenna use (6 bits per channel use), and the results indicate that at the BER of 10^{-3}, the former outperforms the latter by 0.7 dB.

VII. CONCLUSION

In this paper, after summarizing the 2×2 STCs included in the IEEE 802.16e-2005 specifications, we described one full-rate and one rate-3/4 full-diversity 2×2 STCs leading to low-complexity optimum decoding. More specifically, the optimum decoder complexity grows quadratically with the size of the signal constellation for the full rate code and it grows linearly with the size of the signal constellation for the rate-3/4 code. The simulation results indicated that the full-rate STC achieves the performance of the best known code (the Golden code) while reducing the decoder complexity by an order of magnitude in QPSK, two orders of magnitude in 16-QAM, and four orders of magnitude in 64-QAM based MIMO systems. They also showed that the rate-3/4 code is an interesting alternative providing further tradeoffs between performance and spectral efficiency. Thus, the presented STC designs open up new perspectives for future evolutions of mobile WiMAX systems as well as for 3GPP LTE.

REFERENCES

[1] IEEE 802.16-2005: IEEE Standard for Local and Metropolitan Area Networks – *Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems – Amendment 2: Physical Layer and Medium Access Control Layers for Combined Fixed and Mobile Operation in Licensed Bands*, February 2006.

[2] S. M. Alamouti, “A Simple Transmit Diversity Technique for Wireless Communications,” *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[3] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*, Cambridge University Press, 2005.

[4] J.-C. Belfiore, G. Rekaya, and E. Viterbo, “The golden code: a 2 x 2 full-rate space-time code with non-vanishing determinants,” *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1432–1436, April 2005.

[5] S. Sezginer and H. Sari, “Full-rate full-diversity 2×2 space-time codes of reduced decoder complexity,” *IEEE Commun. Lett.*, vol. 11, no. 12, pp. 973–975, December 2007.

[6] S. Sezginer and H. Sari, “A high-rate full-diversity 2×2 space-time code with simple maximum-likelihood decoding,” *Proc. ISSPIT 2007*, December 2007, Cairo, Egypt.

[7] V. Tarokh, N. Seshadri, and A. R. Calderbank, “Space-time codes for high data rate wireless communications: Performance criterion and code construction,” *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, March 1998.

[8] E. Viterbo and J. Boutros, “A universal lattice code decoder for fading channels,” *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1639–1642, July 1999.

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