Full-Rate Full-Diversity 2×2 Space-Time Codes of Reduced Decoder Complexity

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***Abstract*— Multiple-input multiple-output (MIMO) techniques have become an essential part of broadband wireless communications systems. For example, the recently developed IEEE 802.16e specifications for broadband wireless access include three MIMO profiles employing 2×2 space-time codes (STCs) and two of those are mandatory on the downlink of Mobile WiMAX systems. Conventional approaches to STC design are based on performance criteria such as coding gain, diversity gain, multiplexing gain, and ignore the decoder complexity. In this paper, we take an alternative approach and present a full-rate full-diversity 2×2 STC design leading to substantially lower complexity of the optimum detector than existing schemes. This makes the implementation of high performance full-rate codes realistic in practical systems.**

***Index Terms*— Multiple-input multiple-output (MIMO), maximum likelihood (ML) detection, space-time codes (STCs), WiMAX systems.**

I. Introduction

MULTIPLE-input multiple-output (MIMO) techniques based on using multiple antennas at transmitter and receiver can provide spatial diversity, multiplexing gain, interference suppression, and make various tradeoffs between them. These techniques have been incorporated in all of the recently developed wireless communications system specifications including the IEEE 802.11n standard for local area networks and the IEEE 802.16e-2005 standard [1] for mobile broadband wireless access systems. Mobile WiMAX systems are based on the scalable OFDMA mode of the IEEE 802.16e-2005 specifications and use a subset of the different options included in them. Regarding MIMO options, the WiMAX Forum has specified two mandatory profiles for use on the downlink. One of them is based on the space-time code (STC) proposed by Alamouti for transmit diversity [2]. This code provides a diversity order equal to twice the number of antennas at the receiver. But it is only half-rate, because it only transmits two symbols using two time slots and two transmit antennas. The other profile is spatial multiplexing (SM), which uses two transmit antennas to transmit two independent data streams. This scheme is full-rate, but it does not benefit from any diversity gain at the transmitter.

For future evolutions of the WiMAX standard, it is highly desirable to include a new code combining the respective advantages of the Alamouti code and the SM while avoiding their drawbacks. Such a code actually exists in the IEEE 802.16e-2005 specifications (where it is referred to as Matrix C). The Matrix C is a variant of the Golden code [3], which is known to be one of the best 2×2 STCs. However, this code has a tremendous decoding complexity which grows with the fourth power of the signal constellation size, which makes it impractical for low-cost wireless user terminals.

In this paper, we take an alternative approach to STC design and develop a family of 2×2 STCs that are at the same time full-rate, full-diversity, and whose detection complexity grows only quadratically with the size of the signal constellation. Compared to Matrix C (and the Golden code), our approach thus reduces the optimum decoder complexity by a factor of 256 (resp. $4,096$) for the 16-QAM (resp. 64-QAM) signal constellation. First, in the next section, we briefly discuss the general design criteria for STCs. Next, in Section III, we describe the proposed scheme and the corresponding detector. Finally, we present some performance results in Section IV, and give our conclusions in Section V.

II. STC Design Criteria

We start with a brief discussion of the most common design criteria for STCs when the transmitter does not have any channel state information. For a 2×2 transmission, we take

$$\mathbf{X} = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix} \tag{1}$$

as the 2×2 codeword matrix whose elements take values from the codebook $\mathcal{X}$. Code optimization generally relies on the analysis of pairwise error probability (PEP) $P(\mathbf{X} \rightarrow \hat{\mathbf{X}})$ which is the probability that $\hat{\mathbf{X}}$ is detected while $\mathbf{X}$ is transmitted. A Chernoff bound analysis of the PEP was performed in [4], and this analysis led to the following design criteria:

1) Rank Criterion: In order to achieve the maximum diversity, the diversity gain

$$d(\mathcal{X}) = \min_{\substack{\mathbf{X},\hat{\mathbf{X}} \in \mathcal{X} \\ \mathbf{X} \neq \hat{\mathbf{X}}}} \text{rank}[(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H] \tag{2}$$

should be maximized. If $(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H$ is full rank for all codeword pairs, then the code is said to have *full diversity*.

2) Determinant Criterion: The coding gain is defined as

$$\delta = \min_{\substack{\mathbf{X},\hat{\mathbf{X}} \in \mathcal{X} \\ \mathbf{X} \neq \hat{\mathbf{X}}}} \det[(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H]. \tag{3}$$

In order to obtain the best performance, the coding gain should be maximized for a given average transmit power.

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III. Proposed STC

Now, we present our approach to full-rate 2×2 STC design which attempts to maximize both the diversity gain and the coding gain, while leading to an optimum detector of reduced complexity. Precisely, the optimum receiver for this code has a complexity that is only proportional to M^2, where M is the size of the signal constellation.

A group of 4 data symbols (s_1, s_2, s_3, s_4) in the proposed code design is transmitted as follows:

$$\mathbf{X} = \begin{bmatrix} as_1 + bs_3 & -cs_2^* - ds_4^* \\ as_2 + bs_4 & cs_1^* + ds_3^* \end{bmatrix}, \tag{4}$$

where a, b, c, and d are complex-valued design parameters and the star designates complex conjugate.

On the first receive antenna, the two signals received during the first and second symbol intervals are:

$$r_1 = h_{11}(as_1 + bs_3) + h_{12}(as_2 + bs_4) + n_1 \tag{5a}$$
$$r_2 = h_{11}(-cs_2^* - ds_4^*) + h_{12}(cs_1^* + ds_3^*) + n_2. \tag{5b}$$

Similarly, we have on the second receive antenna:

$$r_3 = h_{21}(as_1 + bs_3) + h_{22}(as_2 + bs_4) + n_3 \tag{6a}$$
$$r_4 = h_{21}(-cs_2^* - ds_4^*) + h_{22}(cs_1^* + ds_3^*) + n_4. \tag{6b}$$

In these equations, h_{kl} designates the channel response between transmit antenna l and receive antenna k, and the n_k, $k = 1, \ldots, 4$, terms are circularly symmetric additive Gaussian noise terms with spectral density N_0. The maximum likelihood (ML) detector makes an exhaustive search over all possible values of the transmitted symbols and decides in favor of the quadruplet (s_1, s_2, s_3, s_4) which minimizes the Euclidean distance:

$$\begin{aligned} D(s_1, s_2, s_3, s_4) = \Big\{ & |r_1 - h_{11}(as_1 + bs_3) - h_{12}(as_2 + bs_4)|^2 \\ & + |r_2 + h_{11}(cs_2^* + ds_4^*) - h_{12}(cs_1^* + ds_3^*)|^2 \\ & + |r_3 - h_{21}(as_1 + bs_3) - h_{22}(as_2 + bs_4)|^2 \\ & + |r_4 + h_{21}(cs_2^* + ds_4^*) - h_{22}(cs_1^* + ds_3^*)|^2 \Big\} \end{aligned}$$

An exhaustive search clearly involves the computation of M^4 metrics and $M^4 - 1$ comparisons, which is excessive for the 16-QAM and 64-QAM signal constellations. But the proposed STC design lends itself to a low-complexity implementation of the ML detector as we now show.

From the received signal samples (r_1, r_2, r_3, r_4), let us compute the following intermediate signals

$$z_1 = r_1 - b(h_{11}s_3 + h_{12}s_4) \tag{7a}$$
$$z_2 = r_2 - d(h_{12}s_3^* - h_{11}s_4^*) \tag{7b}$$
$$z_3 = r_3 - b(h_{21}s_3 + h_{22}s_4) \tag{7c}$$
$$z_4 = r_4 - d(h_{22}s_3^* - h_{21}s_4^*) \tag{7d}$$

for a given value of the symbol pair (s_3, s_4). Using (5) and (6), these signals can be expressed as:

$$z_1 = a(h_{11}s_1 + h_{12}s_2) + n_1 \tag{8a}$$
$$z_2 = c(h_{12}s_1^* - h_{11}s_2^*) + n_2 \tag{8b}$$
$$z_3 = a(h_{21}s_1 + h_{22}s_2) + n_3 \tag{8c}$$
$$z_4 = c(h_{22}s_1^* - h_{21}s_2^*) + n_4\ . \tag{8d}$$

Next, from (8a)–(8d), we compute:

$$h_{11}^* z_1 = a(|h_{11}|^2 s_1 + h_{11}^* h_{12} s_2) + h_{11}^* n_1 \tag{9a}$$
$$h_{12} z_2^* = c^*(|h_{12}|^2 s_1 - h_{11}^* h_{12} s_2) + h_{12} n_2^* \tag{9b}$$
$$h_{21}^* z_3 = a(|h_{21}|^2 s_1 + h_{21}^* h_{22} s_2) + h_{21}^* n_3 \tag{9c}$$
$$h_{22} z_4^* = c^*(|h_{22}|^2 s_1 - h_{21}^* h_{22} s_2) + h_{22} n_4^*\ . \tag{9d}$$

Finally, from those signals, we compute the signal y_1 given by:

$$\begin{aligned} y_1 &= (h_{11}^* z_1 + h_{21}^* z_3)/a + (h_{12} z_2^* + h_{22} z_4^*)/c^* \\ &= (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2)s_1 + w_1 \end{aligned} \tag{10}$$

with $w_1 = (h_{11}^* n_1 + h_{21}^* n_3)/a + (h_{12} n_2^* + h_{22} n_4^*)/c^*$.

It can be seen that the signal y_1 has no terms involving symbol s_2, and the coefficient of the term in s_1 clearly indicates that estimation of s_1 benefits from fourth-order spatial diversity. By sending this signal to a threshold detector, we get the ML estimate of symbol s_1 conditional on (s_3, s_4). Note that the elimination of the terms involving s_2 is possible if and only if the respective coefficients of the symbols s_1 and s_2 in each column of the code matrix are identical.

Similarly to (9a)–(9d), we also compute $h_{12}^* z_1$, $h_{11} z_2^*$, $h_{22}^* z_3$, $h_{21} z_4^*$, and then,

$$\begin{aligned} y_2 &= (h_{12}^* z_1 + h_{22}^* z_3)/a - (h_{11} z_2^* + h_{21} z_4^*)/c^* \\ &= (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2)s_2 + w_2 \end{aligned} \tag{11}$$

with $w_2 = (h_{12}^* n_1 + h_{22}^* n_3)/a - (h_{11} n_2^* + h_{21} n_4^*)/c^*$.

Equation (11) shows that signal y_2 has no terms involving symbol s_1 and the coefficient of the term in s_2 indicates that estimation of s_2 benefits from fourth-order spatial diversity. By sending y_2 to a threshold detector, we get the ML estimate of symbol s_2 conditional on (s_3, s_4). In this way, for a given symbol pair (s_3, s_4), we get the ML estimate of (s_1, s_2), which we denote (s_1^{ML}, s_2^{ML}).

Now, instead of computing the metric $D(s_1, s_2, s_3, s_4)$ for all (s_1, s_2, s_3, s_4) values, we only need to compute it for $(s_1^{ML}, s_2^{ML}, s_3, s_4)$, with s_3 and s_4 spanning the signal constellation. Specifically, let (s_3^k, s_4^l) indicate that symbol s_3 takes the kth point of the signal constellation and symbol s_4 takes the lth point of the signal constellation. The optimum receiver computes the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^{ML}, s_2^{ML}, s_3^k, s_4^l)$, where $k, l = 1, 2, \ldots, M$. This procedure reduces the ML receiver complexity from M^4 to M^2.

Note that the special structure of (4) allows the ML detector also to work the other way round: Instead of deriving the ML estimate of (s_1, s_2) conditional on (s_3^k, s_4^l) and then computing the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^{ML}, s_2^{ML}, s_3^k, s_4^l)$, we can first estimate (s_3, s_4) conditional on (s_1^k, s_2^l), then compute the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^k, s_2^l, s_3^{ML}, s_4^{ML})$, $k, l = 1, 2, \ldots, M$, and select the quadruplet (s_1, s_2, s_3, s_4) minimizing the metric.

It is instructive to point out here that the described detector is optimum only when the magnitudes of a and c (alternatively the magnitudes of b and d for the reverse detection order) are equal. This can be easily seen by looking at the signal-to-noise ratio (SNR) at the receiver input and then at the threshold detector input. Indeed, these two SNR values are the same if and only if $|a| = |c|$ for forward detection and $|b| = |d|$ for reverse detection.

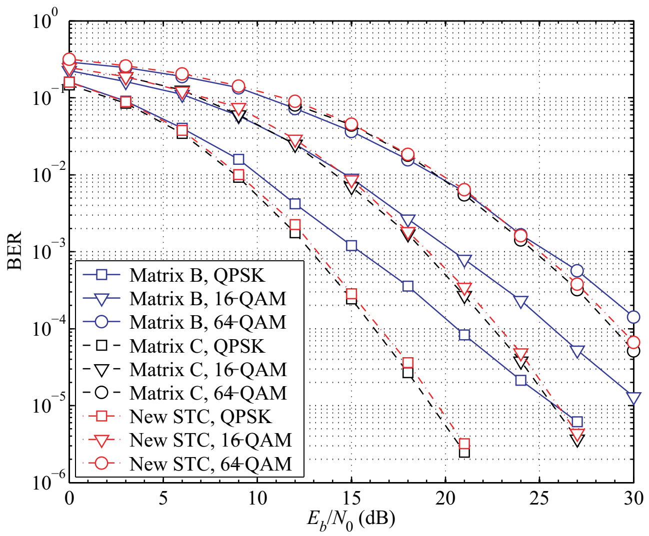

Fig. 1. Performance comparison of the new STC with Matrix B and Matrix C of the IEEE 802.16e-2005 specifications.

The a, b, c, d parameters in the code matrix are design parameters to be optimized in order to obtain full-diversity STC with large coding gain. Below, we utilize the transmit power constraints which lead to further decrease in the number of parameters to be optimized.

In terms of the transmitted power, the desired conditions can be expressed as

$$|a|^2 + |b|^2 = 1 = |c|^2 + |d|^2 \qquad (12)$$

$$|a|^2 + |c|^2 = 1 = |b|^2 + |d|^2. \qquad (13)$$

The first condition ensures an equal transmit power at each symbol time, while the second condition ensures that equal total power is transmitted for each symbol. These equalities together with the constraint $|a| = |c|$ for optimal detection lead immediately to the fact that all the design parameters should have the same magnitude, i.e.,

$$|a| = |b| = |c| = |d| = 1/\sqrt{2}. \qquad (14)$$

Without loss of generality, we take $a = c = 1/\sqrt{2}$ (since they do not affect the coding gain) and choose the b and d parameters which lead to a full-diversity scheme with a large coding gain. According to the rank criterion (cf. Section II), it is evident that in order to obtain full diversity, the difference matrix $(\mathbf{X} - \hat{\mathbf{X}})$ should be full rank. Thanks to the special form of (4), it can be shown that there exists a set of parameters (b, d) for which the resulting STC will have nonzero coding gain for any QAM constellation size. In light of the design criteria given in Section II, this directly ensures that the obtained code will have full diversity for any QAM constellation. However, a generalized optimization procedure of the parameter set is beyond the scope of this letter. Instead, we provide an example for the proposed scheme leading to a large coding gain.

IV. Results

We now provide some performance comparisons between the full-rate 2×2 matrices defined in the IEEE 802.16e-2005 standard and the proposed STC. We set the values of the remaining parameters in (14) as $b = [(1 - \sqrt{7}) + i(1 + \sqrt{7})]/(4\sqrt{2})$ and $d = -ib$, where $i = \sqrt{-1}$. With these parameters, we obtain a coding gain of 2 for all the presented constellation sizes. Fig. 1 shows the bit error rate (BER) performance as a function of E_b/N_0, E_b denoting the average signal energy per bit, and provides comparisons between the proposed scheme, and Matrix B (spatial multiplexing) and Matrix C of WiMAX systems using an uncorrelated Rayleigh fading channel. The results corresponding to Matrix B and the proposed scheme are obtained using the optimum detection for all constellation sizes. Those corresponding to Matrix C are obtained with ML detection for QPSK and ML sphere decoding [5] for the higher constellations. It can be seen that the slopes of the BER curves are much higher for Matrix C and the proposed scheme compared to Matrix B. As a result, these schemes substantially outperform Matrix B at high SNR values. More importantly, the proposed scheme gives essentially the same results as Matrix C at substantially lower complexity.

V. Conclusions

In this paper, we have presented a new full-rate full-diversity 2×2 STC design with an inherent low-complexity optimum decoder. We have compared its performance to those of the STCs included in the IEEE 802.16e-2005 specifications, and the results indicated that the proposed scheme achieves the performance of the best known code while reducing the decoder complexity by an order of magnitude in QPSK, two orders of magnitude in 16-QAM, and four orders of magnitude in 64-QAM based MIMO systems. Thus, the novel STC design opens up new perspectives for future evolutions of WiMAX systems as well as for other broadband wireless systems.

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