

An Asymmetric 2×2 Space-Time Code with Linear Maximum-Likelihood Decoder Complexity

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Abstract – Multiple-input multiple-output (MIMO) techniques have been incorporated in all of the recently developed wireless communications systems standards including the IEEE 802.11n standard for local area networks and the IEEE 802.16e standard on which mobile WiMAX systems are based. The main criteria in conventional space-time code (STC) design for MIMO systems are the code rate, diversity order, and coding gain. Recently, the optimum decoder complexity was added to this list of criteria and the attention was turned to the design of STCs with low decoder complexity. Specifically, the effort in full-rate 2×2 STC design focused on designing codes whose optimum decoder complexity grows only quadratically with the size of the signal constellation, and such a design was proposed by the present authors. Proceeding further, a related STC of rate-3/4 and whose optimum decoder complexity grows only linearly with the constellation size was also proposed. In the present paper, we introduce another interesting high-rate and low complexity 2×2 STC design. The new code has a linear optimum decoding complexity and its rate is higher than 3/4.

Keywords—MIMO systems, space-time codes, WiMAX.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques based on using multiple antennas at transmitter and receiver can provide spatial diversity, multiplexing gain, interference suppression, and make various tradeoffs between them. These techniques have been incorporated in all of the recently developed wireless communications system specifications and have become an indispensable part of the IEEE 802.16e-2005 standard [1] for mobile broadband wireless access systems.

From the MIMO schemes included in the IEEE 802.16e specifications, the WiMAX Forum has specified two mandatory profiles for use on the downlink. One of them is based on the space-time code (STC) proposed by Alamouti for transmit diversity [2]. This code achieves a diversity order that is equal to twice the number of antennas at the receiver, but it is only half-rate, because it only transmits two symbols using two time slots and two transmit antennas. (In this paper, we define the rate as the number of transmitted symbols per antenna use. The Alamouti code is of rate 1 if the rate is defined as the number of symbols per channel use.) The other profile is spatial multiplexing (SM), which uses two transmit antennas to transmit two independent data streams. This scheme is full-rate, but it does not benefit from any diversity gain at the transmitter, and, at best, it provides a diversity order that is equal to the number of receive antennas.

For future evolutions of the WiMAX standard, it is highly desirable to include a new code combining the respective advantages of the Alamouti code (Matrix A) and the SM (Matrix B) while avoiding their drawbacks. Such a code actually exists in the IEEE 802.16e-2005 specifications (where it is referred to as Matrix C). The Matrix C is a variant of the Golden code [3] (see also [4] and [5] for other variants),

which is known to be one of the best 2×2 STCs achieving the diversity-multiplexing frontier [4]. But the problem of this code is its detection complexity, which grows as the fourth-power of the signal constellation size, and this makes it impractical for low-cost wireless user terminals.

Recently, motivated by the orthogonality of Alamouti scheme, new full-rate full-diversity (FR-FD) 2×2 STCs were proposed. These codes achieve the diversity-multiplexing frontier, while their optimum detection complexity (using exhaustive search) grows at most quadratically with the size of the signal constellation (see, e.g., [6] for more detail). In addition, a rate-3/4 code was presented by the present authors in [7] by constraining the FR-FD STC proposed in [6] to have 3 symbols per 2×2 code matrix.

In this paper, we present a new code with linear decoding complexity, which achieves a code rate higher than 3/4. First, in Section II, we briefly recall the FR-FD 2×2 STCs. Next, Section III briefly recalls the rate-3/4 STC proposed in [7]. Then, in Section IV, we describe the proposed asymmetric STC which achieves a code rate higher than 3/4. Finally, we present some numerical comparisons in Section V and give our conclusions in Section VI.

II. FR-FD 2×2 SPACE-TIME CODES

The first STC we describe here is the Matrix C included in the IEEE 802.16-2005 specifications. It is given by

$$\mathbf{X}_C = \frac{1}{\sqrt{1+r^2}} \begin{bmatrix} s_1 + jrs_4 & rs_2 + s_3 \\ s_2 - rs_3 & jrs_1 + s_4 \end{bmatrix}, \quad (1)$$

where $r = (-1 + \sqrt{5})/2$ and $j = \sqrt{-1}$.

The first row of the matrix represents the signals transmitted by the first antenna and the second row those transmitted by the second antenna. The first column represents the signals transmitted by the two transmit antennas during the first channel use and the second column those transmitted by the second channel use. This code is full rate, because it transmits 4 symbols per matrix, or one symbol per antenna use. Also, it leads to a spatial diversity of order 4 for 2 receiver antennas and achieves substantially better performance than the SM code (Matrix B), whose spatial diversity is limited to 2. Moreover, \mathbf{X}_C results in the same bit error probability as the Golden code (and the other variants proposed in [4] and [5]). More specifically, \mathbf{X}_C results in a coding gain of 16/5 which is the largest coding gain obtained so far. \mathbf{X}_C differs from the Golden code and its variants in using a higher order extension field (see, e.g., [8] for related definitions). From the implementation point of view, the only difference is that the construction of \mathbf{X}_C requires a smaller number of multiplications compared to the Golden code.

However, as explained above, the problem of this code is its inherent detection complexity.

The optimum receiver evaluates the maximum likelihood (ML) function for all symbol quadruplets (s_1, s_2, s_3, s_4) and selects the one which maximizes this function. The ML function evaluated for (s_1, s_2, s_3, s_4) is actually the squared Euclidean distance between the received noisy signal and the noiseless signal corresponding to that quadruplet, and can be expressed as the squared Frobenius norm

$$D(s_1, s_2, s_3, s_4) = \|\mathbf{Y} - \mathbf{H}\mathbf{X}\|^2. \quad (2)$$

For a signal constellation with M points, this receiver involves the computation of M^4 Euclidean distances and selects the symbol quadruplet minimizing this distance. The optimum receiver complexity is, therefore, proportional to $16^4 = 65,536$ for a 16-QAM signal constellation, and to $64^4 = 16,777,216$ for a 64-QAM signal constellation. Of course, this complexity is prohibitive for practical applications. Therefore, one usually resorts to suboptimum receivers which may degrade the performance severely.

One possible solution is to use sphere detection (SD) [9] whose performance and complexity are upper bounded by those of ML detection based on exhaustive search. The major issues in the implementation of SD are choosing the initial radius and the order in which the symbols are examined. These two issues can dramatically improve or degrade the complexity of SD. That is, even the use of SD would require a high number of computations for satisfactory detection performance. It is also worth noting that, from a practical point of view, SD suffers from the fact that its throughput is variable. To avoid these problems, there is a significant interest in new STCs which offer performance close to that of \mathbf{X}_C with lower detection complexity.

Now, we present our recently proposed FR-FD 2×2 STC scheme [6], which leads to an optimum detection complexity that is a quadratic function of the number of points of the signal constellation. It reduces the number of Euclidean distance computations in the optimum detector to $16^2 = 256$ for a 16-QAM signal constellation and to $64^2 = 4,096$ for a 64-QAM signal constellation. Comparing these numbers to those associated to Matrix C, we see that this code reduces the optimum decoder complexity by two orders of magnitudes with 16-QAM and four orders of magnitudes with 64-QAM. It is clear that this code makes the implementation of FR-FD 2×2 STCs with optimum receiver more realistic.

Note that a similar STC appears in [10], but its low decoding complexity property was only realized in [11]. This STC is a combination of the original Alamouti scheme and a precoded scheme having also an Alamouti structure. In contrast, our STC has a symmetric structure, as it is a direct combination of two Alamouti schemes. As is shown in the sequel, this allows to evenly distribute the transmitted energy for each symbol per channel use. In our code, the group of 4 symbols (s_1, s_2, s_3, s_4) is transmitted as follows:

$$\mathbf{X}_{full} = \begin{bmatrix} as_1 + bs_3 & -cs_2^* - ds_4^* \\ as_2 + bs_4 & cs_1^* + ds_3^* \end{bmatrix}. \quad (3)$$

A careful look clearly shows that (3) is nothing but an intelligent combination of two Alamouti schemes. Here, a , b , c , and d are complex-valued design parameters. They are

chosen such that the resulting STC attains FR-FD transmission in a quasi-static Rayleigh fading channel. Note that parameter optimization is infeasible especially for higher constellation sizes. In [6], the optimization of the parameters is simplified considering the desired average transmit power constraints. More specifically, we considered

$$|a|^2 + |b|^2 = |c|^2 + |d|^2 = 1 \quad (4)$$

$$|a|^2 + |c|^2 = |b|^2 + |d|^2 = 1 \quad (5)$$

as the transmit power constraints. The first condition ensures the transmission of equal average power at each symbol time, while the second condition ensures that equal average total power is transmitted for each symbol. A simple manipulation can show that the magnitudes of a and c should be equal for reduced complexity optimum detection. These equalities lead immediately to the fact that all the design parameters should have the same magnitude, i.e., $|a| = |b| = |c| = |d| = 1/\sqrt{2}$.

Now, without any loss of generality, we set $a = c = 1/\sqrt{2}$. This decreases the number of unknown parameters without affecting the coding gain. Then, the remaining parameter pair (b, d) can be optimized numerically leading to a full-diversity scheme with large coding gain. Such an optimization has been performed using QPSK signalling and resulted in a set of parameter pairs which gave a coding gain of 2 independent of QAM constellation size. This directly ensures that the obtained code will have full diversity for any QAM constellation size. Moreover, as shown in [4] and [12], such an STC with nonvanishing coding gain achieves the diversity-multiplexing frontier. In the numerical illustrations we provide the example of [6] in which we have $b = [(1 - \sqrt{7}) + j(1 + \sqrt{7})]/(4\sqrt{2})$ and $d = -jb$. We will not repeat here the ML detection steps of this code, which are fully described in [6].

III. RATE-3/4 FULL-DIVERSITY 2×2 STC

The rate-3/4 STC which was introduced in [7] is obtained by setting $s_4 = s_3$ in the coding matrix given by (3) and scaling the power of this symbol. This leads to the following 2×2 code matrix:

$$\mathbf{X}_{3/4} = \begin{bmatrix} as_1 + bs_3/\sqrt{2} & -(cs_2^* + ds_3^*/\sqrt{2}) \\ as_2 + bs_3/\sqrt{2} & cs_1^* + ds_3^*/\sqrt{2} \end{bmatrix}. \quad (6)$$

Following similar steps as in [6], it was shown in [7] that the ML detector of this code has only a linear complexity as a function of the constellation size M . Furthermore, the performance study confirmed that this code represents an interesting tradeoff between the Alamouti code and the FR-FD code presented in the previous section.

In this paper, we will go one step ahead and design full diversity 2×2 STCs with linear ML decoding complexity and higher rates than 3/4.

IV. DESCRIPTION OF THE NEW STC

The code matrix corresponding to the asymmetric STC proposed here is given by:

$$\mathbf{X}_{new} = \begin{bmatrix} as_1 + bs_3 & -\alpha s_2^* \\ as_2 & cs_1^* + ds_3^* \end{bmatrix}. \quad (7)$$

where a, b, c , and d are complex design parameters as in (3), and α is a scaling parameter (real number). For the diagonal elements of the matrix, we take $a = c = 1$, $d = -jb$, and $b = \left[\left((1 + \sqrt{7}) + j(1 - \sqrt{7}) \right) / 4 \right]$ to optimize the coding gain. Furthermore, $\alpha = \sqrt{2}$ if the symbol s_2 takes its values from the same M -QAM signal constellation as s_1 and s_3 .

If we use a single receive antenna, the received signals corresponding to the two channel uses can be written as:

$$r_1 = h_1(s_1 + bs_3) + \alpha h_2 s_2 + n_1 \quad (8.a)$$

and

$$r_2 = -\alpha h_1 s_2^* + h_2(s_1^* - jbs_3^*) + n_2. \quad (8.b)$$

where h_1 and h_2 respectively denote the channel responses from the two transmit antennas to the receive antenna, and n_1 and n_2 are additive noise terms. By subtracting the s_3 terms from these signals, we get:

$$x_1 = r_1 - h_1 bs_3 \quad (9.a)$$

and

$$x_2 = r_2 + jh_2 bs_3^*. \quad (9.b)$$

Using (8.a) and (8.b), these can be written as:

$$x_1 = h_1 s_1 + \alpha h_2 s_2 + n_1 \quad (10.a)$$

and

$$x_2 = -\alpha h_1 s_2^* + h_2 s_1^* + n_2. \quad (10.b)$$

From these two signals, we can compute

$$\begin{aligned} y_1 &= h_1^* x_1 + h_2^* x_2 \\ &= \left(|h_1|^2 + |h_2|^2 \right) s_1 + h_1^* n_2 + h_2^* n_2 \end{aligned} \quad (11)$$

Clearly, by sending this signal to a threshold detector, we get an ML estimate of symbol s_1 conditional on s_3 . We call this $\hat{s}_1^{ML} | s_3$. The diversity order in (11) is 2, but using two receive antennas, repeating the above steps for the signals at the output of the second antennas, and combining the two signals thus obtained, we get a diversity order of 4.

By repeating this process for all points of the signal constellation (possible values of s_3), comparing them, and selecting the conditional estimate $\hat{s}_1^{ML} | s_3$ corresponding to the smallest Euclidean distance, we get the ML estimate of (s_1, s_3) that we denote $(\hat{s}_1^{ML}, \hat{s}_3^{ML})$.

The next steps are as follows:

$$z_1 = r_1 - h_1(\hat{s}_1^{ML} + b\hat{s}_3^{ML}) \quad (12.a)$$

and

$$z_2 = r_2 - h_2(\hat{s}_1^{ML*} - jbs_3^{ML*}). \quad (12.b)$$

Assuming that s_1 and s_3 have been correctly detected, we have

$$z_1 = \alpha h_2 s_2 + n_1 \quad (13.a)$$

and

$$z_2 = -\alpha h_1 s_2^* + n_2. \quad (13.b)$$

Finally, we compute

$$\begin{aligned} y_2 &= h_2^* z_1 - h_1 z_2^* \\ &= \alpha \left(|h_1|^2 + |h_2|^2 \right) s_2 + h_2^* n_1 - h_1 n_2^*. \end{aligned} \quad (14)$$

An ML estimate of s_2 is obtained by sending this signal to a threshold detector after scaling. Suppose now that $\alpha = 1$ and that the signal constellation of the symbol s_2 and that of s_1 and s_3 are scaled in such a way that they have the same minimum distance. In that case, the symbol s_2 should take its values from a $2M$ -QAM signal constellation, when the s_1 and s_3 symbols take their values from an M -QAM signal constellation. Since the average power of the $2M$ -QAM signal constellation is twice that of the M -QAM signal constellation, the same power is transmitted in this case from the two antennas.

The $2M$ -QAM signal constellation transmits $\log_2 2M$ bits per symbol, and similarly, the M -QAM signal constellation transmits $\log_2 M$ bits per symbol. The reference full-rate 2×2 STC transmits 4 M -QAM symbols per code matrix. In our case, two M -QAM symbols and one $2M$ -QAM symbol are transmitted per code matrix. The code rate is therefore $(3m + 1) / 4m$, where $m = \log_2 M$. For $M = 16$, the code rate is $R = 13/16$.

Without much degradation, it was found that the symbol s_2 can also take its values from a $4M$ -QAM signal constellation instead of $2M$ -QAM. Compared to a rate-3/4 STC, this code transmits 2 additional bits per code matrix. The code rate thus becomes $(3m + 2) / 4m$, which leads to a rate-7/8 code for $m = 4$, or equivalently, $M = 16$. But note that this code will have slightly lower performance than the previous case, because the power constraints (same power transmitted from the two antennas) will lead to a constellation minimum distance reduced by 3 dB.

V. PERFORMANCE ANALYSIS

In this section, we provide some performance comparisons between the proposed 2×2 asymmetric STC and the existing alternatives in the IEEE 802.16e specifications. We give two figures showing the bit error rate (BER) performance as a function of E_s / N_0 , E_s denoting the average signal energy per symbol. The results are obtained for an uncorrelated Rayleigh fading channel with $E[|h_{kl}|^2] = 1$ for all k, l . All results correspond to ML performances of the presented STCs.

Fig. 1 provides comparisons between \mathbf{X}_{new} , namely, the new STC, the Alamouti scheme and \mathbf{X}_C (Matrix C). In the latter two, the modulations used lead to a spectral efficiency of 4 bits per antenna use. In the new STC, the symbol transmitted on the diagonal elements of the matrix take their values from the 16-QAM signal constellation, while the symbol on the non-diagonal element takes its values from the 64-QAM signal constellation. In other words, the code rate is 7/8.

The results show that there is a considerable gap between the results corresponding to Matrix C and those corresponding to the Alamouti code. This gap is in excess of 6 dB in the BER range of $10^{-3} - 10^{-4}$. Next, we can see that the results corresponding to the new STC come quite close to those of Matrix C, the deviation being on the order of 0.5 dB at the BER of 10^{-3} and 1.0 dB at the BER of 10^{-4} . On this figure, we have plotted the BER corresponding to the diagonal and non-

diagonal elements of this asymmetric code matrix separately, because they differ in both the constellations used and the way the symbols are detected. Then, the average BER was obtained by taking the mean of these BER values. As it can be observed, the BER values corresponding to the diagonal and non-diagonal elements are virtually the same.

Next, Fig. 2 presents similar results when Matrix C and the new STC employ higher constellation sizes. More specifically, the modulation used here for Matrix C is 64-QAM. (Note that the Alamouti STC would require a 4096-QAM modulation to achieve the same spectral efficiency.) As for the new STC, the symbols on the diagonal elements take their values from 64-QAM, while those on the non-diagonal elements take their values from 256-QAM. It can be seen that \mathbf{X}_{new} gives essentially the same results as \mathbf{X}_C at BER values lower than 10^{-4} . Note however that the code rate in the new code is only 5/6, and therefore the new code will lose 0.8 dB in terms of E_b/N_0 , where E_b is the energy transmitted per bit.

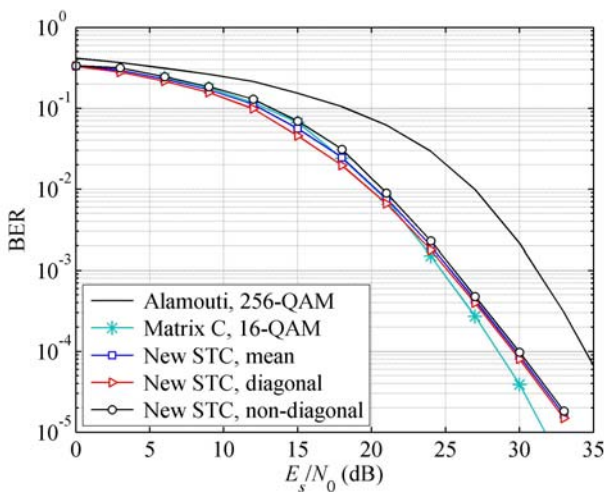


Fig. 1: Performance comparison between Alamouti, \mathbf{X}_C and \mathbf{X}_{new} (4 bits per antenna use in Alamouti and Matrix C, and 3.5 bits per antenna use in the new STC).

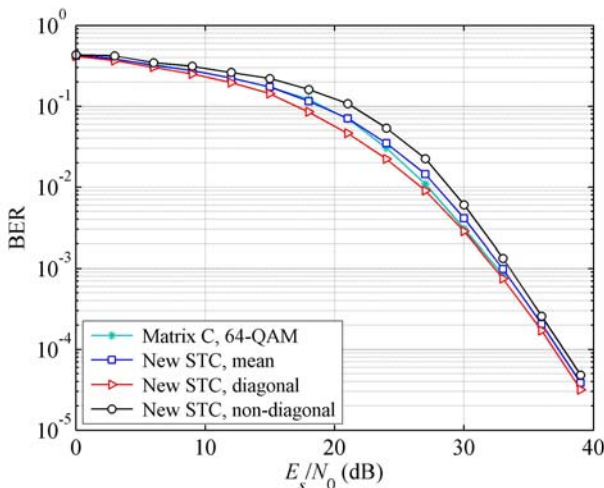


Fig. 2: Performance comparison between \mathbf{X}_C and \mathbf{X}_{new} (6 bits per antenna use in Matrix C and 5 bits per antenna use in the new STC).

VI. CONCLUSIONS

In this paper, we have introduced a new 2×2 STC, which represents an interesting alternative to the STCs included today in the Mobile WiMAX profiles or the IEEE 802.16e – 2005 specifications. The code matrix is asymmetric in the sense that the diagonal and non-diagonal elements transport different numbers of symbols taking their values from different signal constellations. The resulting code rates are less than 1, but they are higher than 3/4. The most interesting feature is that the optimum decoder complexity grows only linearly with the size of the signal constellation, while optimum decoding of Matrix C involves a complexity which grows with the fourth power of the constellation size. It was also found that the new STC comes close to Matrix C in terms of performance and significantly outperforms the Alamouti code. Therefore, the presented asymmetric STC may be of significant interest for future evolutions of mobile WiMAX systems, as well as for other broadband wireless systems.

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