A FULL-RATE FULL-DIVERSITY 2×2 SPACE-TIME CODE FOR MOBILE WiMAX SYSTEMS

Serdar Sezginer[(1)] *and Hikmet Sari*[(1),(2)]

[(1)] Sequans Communications, Bat. Citicenter, 19 Le Parvis, 92073 Paris La Défense, France
[(2)] Supélec, Plateau de Moulon, 3 Rue Joliot Curie, 91190 Gif sur Yvette, France

ABSTRACT

The recently developed IEEE 802.16e specifications for broadband wireless access include three multiple-input multiple-output (MIMO) profiles employing 2×2 space-time codes (STCs) and two of those are mandatory on the downlink of Mobile WiMAX systems. One of these is full rate, and the other is full diversity, but neither of them has both of the desired features. The third profile, which is not mandatory, is both a full-rate and a full-diversity code, but it has a high decoder complexity. In this paper, we develop a full-rate full-diversity STC of size 2×2 which can be optimally detected at a substantially lower complexity than conventional schemes. This makes it a suitable candidate for future evolutions of WiMAX systems toward higher data rates and performance.

Index Terms**—** MIMO systems; ML detection; space-time codes; WiMAX

1. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques based on using multiple antennas at both transmitter and receiver can provide spatial diversity, multiplexing gain, interference suppression, and make various tradeoffs between them. These techniques have been incorporated in all of the recently developed wireless communications system specifications including the IEEE 802.11n standard for local area networks and the IEEE 802.16e-2005 standard [1] for mobile broadband wireless access systems. Mobile WiMAX systems are based on the scalable OFDMA mode of IEEE 802.16e-2005 specifications and use a subset of the different options included in them. Regarding MIMO options, the WiMAX Forum has specified two mandatory profiles for use on the downlink. One of them is based on the space–time code (STC) proposed by Alamouti for transmit diversity [2]. This code provides perfect second-order diversity when used with a single receive antenna and fourth-order diversity when used with two antennas at the receiver. But it is only half-rate, because it only transmits two symbols using two time slots and two transmit antennas. The other profile is spatial multiplexing (SM), which uses two transmit antennas to transmit two independent data streams. This scheme is full-rate, but it does not benefit from any diversity gain at the transmitter, and at best, it provides second-order diversity with two antennas at the receiver.

For future evolutions of the WiMAX standard, it is highly desirable to include a new code combining the respective advantages of the Alamouti code and the SM while avoiding their drawbacks. Such a code actually exists in the IEEE 802.16e-2005 specifications, where it is referred to as Matrix C. The matrix C is a variant of the Golden code [3], which is known to be one of the best STCs of size 2×2. However, this code has a tremendous decoding complexity which grows with the fourth power of the signal constellation size, and this makes it impractical for low-cost wireless systems.

In this paper, we develop a 2×2 STC which is both full-rate and full-diversity, and (at the same time) whose detection complexity grows only quadratically with the size of the signal constellation. Compared to Matrix C (and to the Golden code), our approach thus reduces the optimum decoder complexity by a factor of 256 (resp. 4,096) for the 16-QAM (resp. 64-QAM) signal constellation.

The rest of the paper is organized as follows. First, in the next section, we briefly discuss the general design criteria for STCs. Next, in Section 3, we describe the proposed STC and discuss its advantages. Finally, we present some performance results in Section 4, and give our conclusions in Section 5.

2. STC DESIGN CRITERIA

We start with a brief discussion of the most common design criteria for STCs when the transmitter does not have any channel state information. For the 2×2 transmission, we write

$$\mathbf{R} = \mathbf{H}\mathbf{X} + \mathbf{Z}, \tag{1}$$

where $\mathbf{H}$ is the 2×2 channel matrix with the entries of complex channel gains, $\mathbf{X}$ is the 2×2 codeword matrix

$$\mathbf{X} = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix}. \tag{2}$$

In (1), $\mathbf{R}$ includes the received signals, and $\mathbf{Z}$ denotes the matrix of additive noise samples. Code optimization generally relies on the analysis of pairwise error probability (PEP) $P(\mathbf{X} \rightarrow \hat{\mathbf{X}})$ which is the probability that

$\hat{\mathbf{X}}$ is detected while $\mathbf{X}$ is transmitted. The goal is to minimize the error probability, but the analysis of exact error performance is difficult, and therefore one resorts to the union bound

$$P_s \leq \sum_{\mathbf{X}\in X}\sum_{\hat{\mathbf{X}}\neq\mathbf{X}} P(\mathbf{X}\rightarrow\hat{\mathbf{X}}) , \qquad (3)$$

where X denotes the codebook of the space-time code. A Chernoff bound analysis of the PEP was performed in [4], and this analysis led to the following design criteria:

1) Rank Criterion: In order to achieve the maximum diversity, the diversity gain

$$d(X) = \min_{\substack{\mathbf{X},\hat{\mathbf{X}}\in X \\ \mathbf{X}\neq\hat{\mathbf{X}}}} \mathrm{rank}(\Delta\Delta^H) , \qquad (4)$$

where $\Delta = \mathbf{X}-\hat{\mathbf{X}}$, should be maximized. If $\Delta\Delta^H$ is full rank for all code word pairs, then the code is said to have *full diversity*.

2) Determinant Criterion: The coding gain is defined as

$$\delta = \min_{\substack{\mathbf{X},\hat{\mathbf{X}}\in X \\ \mathbf{X}\neq\hat{\mathbf{X}}}} \prod_{l=1}^{d(X)} \lambda_l(\Delta\Delta^H) , \qquad (5)$$

where $\lambda_l(\cdot)$ denotes the lth largest eigenvalue of the enclosed matrix. In order to obtain the best performance, the coding gain should be maximized for a given average transmit power.

It should be noted that, for high signal-to-noise ratio (SNR) values, the most important parameter is the diversity gain which dominates the steepness of the bit-error rate (BER) curve. Afterwards, it is the coding gain which should be maximized. In the sequel, we will directly optimize the proposed code such that the coding gain is maximized. In light of the above design criteria, this directly ensures that the optimized code will have full diversity.

3. PROPOSED STC

Now, we present our approach to full-rate 2×2 STC design which attempts to maximize both the diversity gain and the coding gain, while leading to an optimum detector of reduced complexity. More specifically, the proposed STC is a full-rate, full-diversity 2×2 space-time code whose optimum receiver has a complexity that is *only* proportional to M^2, where M is the size of the signal constellation. Thus, the number of Euclidean distance computations in the optimum detector is reduced to 16^2 = 256 for a 16-QAM signal constellation and to 64^2 = 4,096 for a 64-QAM signal constellation. Comparing these numbers to those associated to the Golden code (or Matrix C), it becomes clear that this code makes the implementation of full-rate, full-diversity 2×2 STCs with optimum receiver realistic. We now give a general description of the proposed code.

A group of 4 data symbols (s_1, s_2, s_3, s_4) in the proposed code design is transmitted as follows:

$$\mathbf{X} = \begin{bmatrix} as_1 + bs_3 & -cs_2^* - ds_4^* \\ as_2 + bs_4 & cs_1^* + ds_3^* \end{bmatrix} , \qquad (6)$$

where a, b, c, and d are complex-valued design parameters and the star designates complex conjugate.

In this matrix representation, the first column represents the symbol combinations transmitted during a first symbol interval t_1 and the second column represents the symbol combinations transmitted during a second symbol interval t_2. The first row of the matrix gives the symbol combinations transmitted from the first Tx antenna, and second row of the matrix gives the symbol combinations transmitted from the second Tx antenna. In other words, $as_1 + bs_3$ is transmitted from Tx antenna 1 during the first symbol interval t_1, $as_2 + bs_4$ is transmitted from Tx antenna 2 during the first symbol interval t_1, $-cs_2^* - ds_4^*$ is transmitted from Tx antenna 1 during the second symbol interval t_2, and $cs_1^* + ds_3^*$ is transmitted from Tx antenna 2 during the second symbol interval t_2.

On the first receive antenna, the two signals received at the first and second symbol intervals are:

$$r_1 = h_{11}(as_1 + bs_3) + h_{12}(as_2 + bs_4) + n_1 \qquad (7.a)$$

$$r_2 = h_{11}(-cs_2^* - ds_4^*) + h_{12}(cs_1^* + ds_3^*) + n_2 . \qquad (7.b)$$

Similarly, we have on the second Rx antenna:

$$r_3 = h_{21}(as_1 + bs_3) + h_{22}(as_2 + bs_4) + n_3 \qquad (8.a)$$

$$r_4 = h_{21}(-cs_2^* - ds_4^*) + h_{22}(cs_1^* + ds_3^*) + n_4 , \qquad (8.b)$$

where n_i, for $i = 1,\ldots,4$, are the additive noise terms.

The maximum likelihood (ML) detector makes an exhaustive search over all possible values of the transmitted symbols and decides in favor of the quadruplet (s_1, s_2, s_3, s_4) which minimizes the Euclidean distance:

$$\begin{aligned} D(s_1,s_2,s_3,s_4) = \Big\{ & |r_1 - h_{11}(as_1 + bs_3) - h_{12}(as_2 + bs_4)|^2 \\ & + |r_2 - h_{11}(-cs_2^* - ds_4^*) - h_{12}(cs_1^* + ds_3^*)|^2 \\ & + |r_3 - h_{21}(as_1 + bs_3) - h_{22}(as_2 + bs_4)|^2 \\ & + |r_4 - h_{21}(-cs_2^* - ds_4^*) - h_{22}(cs_1^* + ds_3^*)|^2 \Big\} \end{aligned}$$

An exhaustive search clearly involves the computation of M^4 metrics and M^4-1 comparisons, which is excessive for the 16-QAM and 64-QAM signal constellations. But the proposed STC design lends itself to a low-complexity implementation of the ML detector as we now show.

From the received signal samples (r_1, r_2, r_3, r_4), let us compute the following signals:

$$w_1 = r_1 - b(h_{11}s_3 + h_{12}s_4) = a(h_{11}s_1 + h_{12}s_2) + n_1 \qquad (9.a)$$

$$w_2 = r_2 - d(h_{12}s_3^* - h_{11}s_4^*) = c(h_{12}s_1^* - h_{11}s_2^*) + n_2 \qquad (9.b)$$

$$w_3 = r_3 - b(h_{21}s_3 + h_{22}s_4) = a(h_{21}s_1 + h_{22}s_2) + n_3 \quad (9.c)$$
$$w_4 = r_4 - d(h_{22}s_3^* - h_{21}s_4^*) = c(h_{22}s_1^* - h_{21}s_2^*) + n_4 . \quad (9.d)$$

Next, from (w_1, w_2, w_3, w_4), we compute:

$$h_{11}^* w_1 = a(|h_{11}|^2 s_1 + h_{11}^* h_{12} s_2) + h_{11}^* n_1 \quad (10.a)$$
$$h_{12} w_2^* = c^*(|h_{12}|^2 s_1 - h_{11}^* h_{12} s_2) + h_{12} n_2^* \quad (10.b)$$
$$h_{21}^* w_3 = a(|h_{21}|^2 s_1 + h_{21}^* h_{22} s_2) + h_{21}^* n_3 \quad (10.c)$$
$$h_{22} w_4^* = c^*(|h_{22}|^2 s_1 - h_{21}^* h_{22} s_2) + h_{22} n_4^* . \quad (10.d)$$

From those signals, we next compute the signal y_1 given by:

$$y_1 = (h_{11}^* w_1 + h_{21}^* w_3)/a + (h_{12} w_2^* + h_{22} w_4^*)/c^*$$
$$= (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2) s_1 + \eta_1 \quad (11)$$

with $\eta_1 = (h_{11}^* n_1 + h_{21}^* n_3)/a + (h_{12} n_2^* + h_{22} n_4^*)/c^*$.

It can be seen that the signal y_1 has no terms involving symbol s_2, and the coefficient of the term in s_1 clearly indicates that estimation of s_1 benefits from 4th-order spatial diversity. By sending this signal to a threshold detector, we get the ML estimate of symbol s_1 conditional on (s_3, s_4). Note that the elimination of the terms involving s_2 is possible if and only if the respective coefficients of the symbols s_1 and s_2 in each column of the code matrix are identical.

Similarly, we compute the intermediate signals $h_{12}^* w_1$, $h_{11} w_2^*$, $h_{22}^* w_3$, $h_{21} w_4^*$, and then,

$$y_2 = (h_{12}^* w_1 + h_{22}^* w_3)/a - (h_{11} w_2^* + h_{21} w_4^*)/c^*$$
$$= (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2) s_2 + \eta_2 \quad (12)$$

with $\eta_2 = (h_{12}^* n_1 + h_{22}^* n_3)/a - (h_{11} n_2^* + h_{21} n_4^*)/c^*$.

As previously, signal y_2 has no terms involving symbol s_1 and the coefficient of the term in s_2 shows that estimation of s_2 benefits from 4th-order spatial diversity. By sending y_2 to a threshold detector, we get the ML estimate of symbol s_2 conditional on (s_3, s_4). ML estimation of s_1 and s_2 conditional on (s_3, s_4) is illustrated in Figure 1. In this way, for a given symbol pair (s_3, s_4), we get the ML estimate of (s_1, s_2), which we denote (s_1^{ML}, s_2^{ML}). Now, instead of computing the metric $D(s_1, s_2, s_3, s_4)$ for all (s_1, s_2, s_3, s_4) values, we only need to compute it for $(s_1^{ML}, s_2^{ML}, s_3, s_4)$, with s_3 and s_4 spanning the signal constellation. Specifically, let (s_3^k, s_4^l) indicate that symbol s_3 takes the kth point of the signal constellation and symbol s_4 takes the lth point of the signal constellation. The optimum receiver computes the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^{ML}, s_2^{ML}, s_3^k, s_4^l)$, where k, l = 1, 2, …, M. This procedure, which is illustrated in Figure 2, reduces the ML receiver complexity from M^4 to M^2.

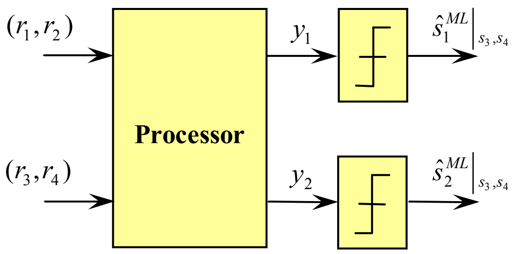

Figure 1. Processing of the received signals to determine the ML estimate of symbols s_1 and s_2 conditional on a particular combination of symbols s_3 and s_4.

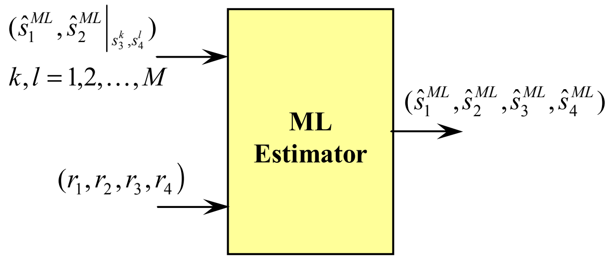

Figure 2. Second stage of the estimator.

Note that the special structure of (6) allows the ML detector also to work the other way round: Instead of deriving the ML estimate of (s_1, s_2) conditional on (s_3^k, s_4^l) and then computing the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^{ML}, s_2^{ML}, s_3^k, s_4^l)$, we can first estimate (s_3, s_4) conditional on (s_1^k, s_2^l), then compute the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^k, s_2^l, s_3^{ML}, s_4^{ML})$, k, l = 1, 2, …., M, and select the quadruplet (s_1, s_2, s_3, s_4) minimizing the metric.

It is instructive to point out here that the described detector is optimum only when the magnitudes of a and c (alternatively the magnitudes of b and d for the reverse detection order) are equal. This can be easily seen by looking at the SNR at the receiver input and then at the threshold detector input. Indeed, these two SNR values are the same if and only if $|a| = |c|$ for forward detection and $|b| = |d|$ for reverse detection.

The a, b, c, d parameters in the code matrix are design parameters to be optimized in order to obtain full-diversity STC with large coding gain. However, this task is infeasible especially for higher constellation sizes. Fortunately, the transmit power constraints can further decrease the number of parameters to be optimized.

In terms of the transmitted power, the desired conditions can be expressed as

$$|a|^2 + |b|^2 = 1 = |c|^2 + |d|^2$$
$$|a|^2 + |c|^2 = 1 = |b|^2 + |d|^2 .$$

The first condition ensures an equal transmit power at each symbol time, while the second condition ensures that equal total power is transmitted for each symbol. These equalities together with the constraint $|a| = |c|$ for optimal detection lead immediately to the fact that all the design parameters should have the same magnitude, *i.e.*,

$$|a| = |b| = |c| = |d| = 1/\sqrt{2} .$$

Without any loss of generality, we take $a = c = 1/\sqrt{2}$ (this allows to decrease the number of unknown parameters without affecting the coding gain) and make an exhaustive search to optimize the parameters *b* and *d*.

Remark: Since the number of Rx antennas are set to the minimum value of two in the current IEEE 802.16e-2005 specifications, the receiver description given above is made for two Rx antennas. However, the presented method can also be generalized for MIMO systems having any number N_R of Rx antennas. In this case, it can be easily seen that the estimation of the symbols benefits from full spatial diversity of order $2N_R$.

4. RESULTS

We now provide some performance comparisons between the full-rate 2×2 matrices defined in the IEEE 802.16e-2005 standard and the proposed STC. To set the values of the *b* and *d* parameters in (6), we have performed an exhaustive search so as to maximize the coding gain (and, thus, to ensure the full diversity) for QPSK signaling. This optimization leads to a set of parameter values which result in a coding gain of 2 for all the presented constellation sizes. From this set we take the parameter pair as $b = \left[(1-\sqrt{7}) + i(1+\sqrt{7})\right]/(4\sqrt{2})$ and $d = e^{-i\pi/2} b$.

Figure 3 shows the BER performance comparisons between the proposed scheme, and Matrix B (SM) and Matrix C of IEEE 802.16e-2005 specifications using an uncorrelated Rayleigh fading channel as a function of E_b / N_0, E_b denoting the average signal energy per bit and N_0 denoting the noise spectral density. The results corresponding to Matrix B and the proposed scheme are obtained using the optimum detection for all constellation sizes. Those corresponding to Matrix C are obtained with ML detection for QPSK and sphere decoding [5] for the higher constellations. It can be seen that the slopes of the BER curves are much higher for Matrix C and the proposed scheme compared to Matrix B. As a result, these schemes substantially outperform Matrix B at high SNR values. More importantly, the proposed scheme gives essentially the same results as Matrix C at substantially lower complexity.

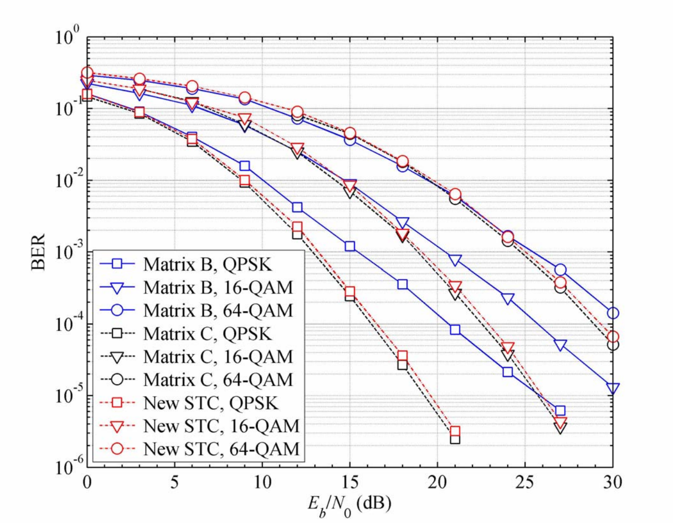

Figure 3. Performance comparison of the new STC with Matrix B and Matrix C of the IEEE 802.16e-2005 specifications.

5. CONCLUSIONS

In this paper, we have presented a new full-rate full-diversity 2×2 STC leading to an inherent low-complexity optimum decoder. We have compared its performance to those of the STCs included in the IEEE 802.16e-2005 specifications, and the results indicated that the proposed scheme achieves the performance of the best known code while reducing the decoder complexity by an order of magnitude in QPSK, two orders of magnitude in 16-QAM, and four orders of magnitude in 64-QAM based MIMO systems. Thus, the novel STC design opens up new perspectives for future evolutions of WiMAX systems, as well as for other wireless systems.

6. REFERENCES

[1] IEEE 802.16-2005: IEEE Standard for Local and Metropolitan Area Networks – *Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems – Amendment 2: Physical Layer and Medium Access Control Layers for Combined Fixed and Mobile Operation in Licensed Bands*, Feb. 2006.

[2] S. M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[3] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden code: A 2 x 2 full-rate space-time code with non-vanishing determinants," *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1432–1436, Apr. 2005.

[4] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.

[5] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1639–1642, Jul. 1999.

[6] H. Yao and G. W. Wornell, "Achieving the full MIMO diversity-multiplexing frontier with rotation-based space-time codes," in *Proc. Allerton Conf. Commun., Control, and Computing*, Monticello, IL, Oct. 2003.

[7] P. Dayal and M. K. Varanasi, "An optimal two transmit antenna spacetime code and its stacked extensions," in *Proc. Asilomar Conf. on Signals, Systems and Computers*, Monterey, CA, Nov. 2003, pp. 987–991.