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OFDM Peak Power Reduction Techniques Performance Analysis for WiMAX Systems
Presentation Outline

- The peak power problem in OFDM
- Peak power reduction techniques
- Methods based on constellation extension or predistortion
- Discussion on performance evaluation criteria
- Simulations using typical power amplifier models and spectral masks for WiMAX systems
- Summary and conclusions
Introduction

- It is well known that OFDM signals have a large peak-to-average power ratio (PAPR), which leads to an inefficient use of the high-power amplifier (HPA) used at the transmitter.

- Indeed, an OFDM signal is the sum of a multitude of digitally-modulated signals with a Gaussian-like probability density function.

- As a result, the peak power of an OFDM signal is much higher than that of a single-carrier signal based on the same modulation and offering a similar spectral efficiency.
The Peak Power Problem

- OFDM is characterized by an inverse fast Fourier transform (IFFT) at the transmitter. The IFFT output block is related to the input symbol block through the relation:

\[ b^i = (b_0^i, b_1^i, \ldots, b_{N-1}^i) \]
\[ a^i = (a_0^i, a_1^i, \ldots, a_{N-1}^i) \]

\[ b_n^i = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} a_m^i e^{\frac{j2\pi mn}{N}} \]

- The peak-to-average power ratio (PAPR) at the IFFT output is given by:

\[ PAPR(b) = \max_{0 \leq n \leq N-1} \frac{|b_n|^2}{E\left\{ \|b\|^2 \right\} / N} \]

- The PAPR of the transmitted signal is obtained by interpolating the IFFT output by a factor of 4.
OFDM peak power reduction has been a hot topic for over a decade and number of techniques have appeared in the literature.

They all aim at reducing the HPA back-off and the effects of its nonlinearity.

Examples are coding, selective mapping (SLM), partial transmit sequences (PTS), and tone reservation. These techniques either reduce the data rate or require the transmission of side information to the receiver.

More recently, some techniques were introduced which do not require the transmission of any side information.
Principle of the ACE Method

- Symbol Mapping
- IFFT
- Clipping
- FFT
- Constellation Constraints
ACE-POCS Algorithm

- The inverse FFT of the input block a is computed.

- At the IFFT output, the peaks exceeding a threshold A are clipped.

- The forward FFT of the clipped output signal block is computed.

- In the resulting symbol block, the distorted symbol values which fall in the undesired regions of the constellation diagram are restored as in the following figure.

- The process is iterated until the desired peak power reduction is achieved or a predetermined number of iterations are carried out.
Illustration of the ACE for 16-QAM
An OFDM signal with 256 carriers composed of 192 data carriers, 8 pilot carriers, 1 DC and 55 guard carriers. No coding is employed.

Rapp’s solid-state power amplifier (SSPA) model was used. It is characterized by the input/output relationship:

\[
\frac{v_{OUT}}{v_{IN}} = \frac{1}{1 + \left(\frac{|v_{IN}|}{v_{SAT}}\right)^2}^{1/p}
\]

The \(p\) parameter in this expression controls the smoothness of the SSPA characteristic. We use here \(p = 2\), which leads to a good representation of the HPA’s used in the sub-10 GHz frequency range.
In the literature, the complementary cumulative distribution function (CCDF) of the peak-to-average power ratio (PAPR) is often used as a criterion.

This function is defined as the probability that the PAPR of transmitted OFDM symbols exceeds a threshold value $\gamma^2$.

$$\text{CCDF}(PAPR(b)) = \Pr(PAPR(b) > \gamma^2)$$
PAPR Improvement with POCS

![Graph showing PAPR improvement with POCS]

- **Pr (PAPR per symbol > γ²)**
- **γ² (dB)**

Legend:
- Initial PAPRs
- 10th POCS iteration, QPSK
- 10th POCS iteration, 16QAM
- 10th POCS iteration, 64QAM
Spectrum with QPSK and Ideal HPA

- Power Spectrum (dB/Hz)

- Frequency (Hz) x 10^6

- No PAPR reduction, clipped, IBO=OBO=9dB

- 10 ACE-POCS iterations, clipped, IBO=OBO=6.7dB

- Original OFDM signal
Spectrum with QPSK and Rapp-Model HPA

- No PAPR reduction, after HPA
  - IBO=9dB, OBO=9.2dB

- 10 ACE-POCS iterations, after HPA
  - IBO=8dBdB, OBO=8.2dB

Original OFDM signal

Power Spectrum (dB/Hz)

Frequency (Hz)
Interpretation of the Results

Assume that the average transmit signal power is backed off from saturation by 9 dB. In the absence of peak power reduction, the signal is clipped with a probability of $2 \times 10^{-1}$.

With the POCS algorithm, the HPA back-off can be reduced to 6.8 dB while preserving the same clipping probability.

In the Rapp-model HPA with $p = 2$, the spectrum is affected not only by clipping, but essentially by all signal samples falling in the nonlinear region.

The gain in this case is much smaller than that predicted by the CCDF of PAPR.
Introducing a New Performance Criterion

- The PAPR CCDF takes into account only the peak value of the output signal, whereas the degradation is caused by all signal samples falling in the nonlinear region of the HPA.

- It is therefore more significant to analyze the CCDF of the instantaneous normalized signal power (INP) defined as:

$$\text{CCDF}(\text{INP}(v_{IN})) = \Pr \left( \frac{|b_i|^2}{P_{avg,IN}} > \gamma^2 \right)$$

- This function gives a much more accurate estimate of the gain that can be achieved using a peak power reduction technique.
CCDF of INP with the POCS Algorithm

\[ P_r (\text{instantaneous normalized signal power} > \gamma^2) \]

- Initial INPs
- 10th POCS iteration, QPSK
- 10th POCS iteration, 16QAM
- 10th POCS iteration, 64QAM

\[ \gamma^2 \text{ (dB)} \]

\[ 10^{-7} \quad 10^{-6} \quad 10^{-5} \quad 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \]
Compatibility with ETSI Mask Type E

ETSI mask type E (QPSK)

- no PAPR reduction, after HPA
  - IBO=4.4dB, OBO=5.3dB
- 10 ACE-POCS iterations, after HPA
  - IBO=3.7dB, OBO=4.6dB

Power Spectrum (dB/Hz)

Frequency (Hz) x 10^6

original OFDM signal
Compatibility with ETSI Mask Type G

ETSI mask type G
no PAPR reduction, after HPA
IBO=7.4dB, OBO=7.8dB

10 ACE-POCS iterations, after HPA
IBO=6.55dB, OBO=6.9dB

Power Spectrum (dB/Hz)

Frequency (Hz)
SNR Degradation for QPSK and Ideal HPA
SNR Degradation, QPSK & Rapp-Model $p=2$
Summary and Conclusions

- We have evaluated the effectiveness of recently proposed OFDM peak power reduction techniques using the IEEE 802.16d parameters along with usual HPA models and several spectral masks.

- The conventional performance criterion (CCDF of the PAPR) was found to be inaccurate in predicting the system gain in terms of total SNR degradation, and a new criterion was proposed.

- It was found that the effective gain of peak power reduction techniques is quite modest due to the tight spectral masks used.

- Other techniques (such as HPA linearization) need to combined to reduce transmit amplifier back-off.